Examples of Tasks from
CCSS Edition Course 2, Unit 2

Getting Started

The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in English and Spanish, is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance. Comments in red type are not part of the solution.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Discrete Mathematics page might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 2

Upon completion of this unit, students should be able to:

- see the interconnectedness of mathematics through use of matrices to solve problems in algebra, geometry, statistics, and discrete mathematics.
- use matrices to organize, display, and analyze data from a variety of contexts, such as archeology, sociology, ecology, sports, and business.
- understand, carry out, and interpret matrix operations—row and column sums, matrix addition and subtraction, scalar multiplication (multiply a matrix by a number), and matrix multiplication.
- understand and apply properties of matrices and matrix operations, compare properties of matrices to those of real numbers, and thereby gain a gentle introduction to algebraic structure.
- use matrices to solve systems of two linear equations.
- compare and analyze different methods for solving systems of two linear equations, by considering limitations, advantages, and disadvantages of methods learned in this and prior units.

What Solutions are Available?

Lesson 1: Investigation 1—Applications Task 2 (p. 88), Connections Task 7 (p. 92), Review Task 23 (p. 100), Review Task 25 (p. 101)
Investigation 2—Extensions Task 19 (p. 97)
Investigation 3—Applications Task 4 (p. 89), Extensions Task 21 (p. 99), Review Task 30 (p. 102), Review Task 31 (p. 102)

Lesson 2: Investigation 1—Applications Task 2 (p. 119), Review Task 24 (p. 130), Review Task 26 (p. 130)
Investigation 2—Applications Task 3 (p. 119), Reflections Task 13 (p. 125), Review Task 32 (p. 131)
Investigation 3—Applications Task 5 (p. 120), Reflections Task 14 (p. 125)

Lesson 3: Investigation 1—Applications Task 2 (p. 145), Connections Task 8 (p. 148), Review Task 23 (p. 155)
Investigation 2—Applications Task 4 (p. 146), Reflections Task 13 (p. 151)
Investigation 3—Connections Task 10 (p. 149), Review Task 28 (p. 156)

Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the CCSS Edition book.
For homework tasks in books with earlier copyright dates, see Helping with Homework.)

Lesson 1, Investigation 1, Applications Task 2 (p. 88)

a. **i.** No, students A and D are not mutual friends because D does not consider A a friend.

ii–iii. To be completed by the student.

b. **Mutual Friends**

	A	B	C	D	E
A	0	0	1	0	1
B	0	0	0	1	0
C	1	0	0	1	0
D	0	1	1	0	1
E	1	0	0	1	0

c–d. To be completed by the student.

Lesson 1, Investigation 1, Connections Task 7 (p. 92)

a. $50.833 \approx 51$ pairs

b. Reebok has the most variability, but students need to determine the variability of each row and compare them. (Possible measures are the standard deviation, the interquartile range, and the range of values.)

c–d. To be completed by the student.

Lesson 1, Investigation 1, Review Task 23 (p. 100)

a. Exponential; $y = 0.25(4^x)$; $NEXT = 4NOW$

b–d. To be completed by the student.

Lesson 1, Investigation 1, Review Task 25 (p. 101)

a. $x = \pm 6$

b. $x = 5$

c. $x = 80$

d–f. To be completed by the student.

Lesson 1, Investigation 2, Extensions Task 19 (p. 97)

a. For the people that are in their second year of freedom, there are only two choices for the next year: (1) third year of freedom or (2) back in jail.

b. Looking at the title, we see that the columns represent next year, so 88% of the people released two years ago will enter their third year of freedom.

c–e. To be completed by the student.

Lesson 1, Investigation 3, Applications Task 4 (p. 89)

Students do not typically have much difficulty reading information from a matrix that has already been constructed. They have more trouble constructing a matrix themselves, or manipulating a matrix to create more information. It is important that they label the rows and columns of the matrices they make, to help them make sense of the result.

a.

October

$O =$

	15-in.	16-in.	17-in.
Chrome	16	24	8
Silver	8	12	4

November

$N =$

	15-in.	16-in.	17-in.
Chrome	12	32	16
Silver	12	20	0

b. To combine the two months' orders we have to *add* the matrices.

October–November Order

	15-in.	16-in.	17-in.
Chrome	28	56	24
Silver	20	32	4

c. **i.** To be completed by the student.

ii. The "–4" entry makes no sense. It must mean that the retailer had overstocked chrome 16-inch wheels. Therefore, the retailer would not order any chrome 16-inch wheels in December.

d. This requires a **scalar multiplication** of the Monthly matrices.

2O

	15-in.	16-in.	17-in.
Chrome	32	48	16
Silver	16	24	8

The remainder of the solution is left for the student to complete.

Lesson 1, Investigation 3, Extensions Task 21 (p. 99)

a. **i.** Both the 0% entry in the 6th row and 1st column, and the 100% entry in the 6th row and 6th column correspond to this assumption because we are assuming that *none* of the persons who have remained out of prison for more than 4 years will be back in prison the next year, so *all* of them will still be free.

ii. The trend from the first 5 rows of the matrix indicates that the percentage who return to prison declines as the number of years of freedom increases. If this continues, then the percentage who return to prison after 4 years will be very small, though probably not 0%. The four-year assumption indicated by the 6th row was presumably made to represent the end point of this trend, and to make the matrix have a manageable number of rows.

b. A released prisoner who is in the second year of freedom is more likely to go back to jail than a released prisoner in the fourth year of freedom.

c. When released from prison, they enter their first year of freedom. And 81% of all prisoners who are in their first year of freedom remain free the next year. Thus, 81% of all those released remain free for more than one year, and they enter their second year of freedom. 88% of those in their second year of freedom are still free the next year. Thus, 88% of 81%, or 71%, of all prisoners released remain free for more than two years (and enter their third year of freedom). Of this 71% of all released prisoners, only 93% are still free the following year. Thus, 66% (0.93×0.71) of all released prisoners remain free for more than three years.

d. About 34% of all released prisoners are sent back to prison within 3 years (100% minus the 66% who are still free). Another way to get this answer would be to total the percentage which is sent back to prison each year. Thus, 19% of all released will be sent back to prison after 1st year, 12% of 81% will be sent back the next year, 7% of 88% of 81% will be sent back after 3 years.
So, $19\% + (0.12 \cdot 81\%) + (0.07 \cdot 0.88 \cdot 81\%) = 33.7\ \%$.

e–f. To be completed by the student.

Lesson 1, Investigation 3, Review Task 30 (p. 102)

a. $10x^5y^6$

b. $\frac{3y^4}{x^2}$ or $3x^{-2}y^4$

c–e. To be completed by the student.

Lesson 1, Investigation 3, Review Task 31 (p. 102)

a. $25x^3 - 30x + 100 = \underline{\ 5\ }(5x^2 + \underline{\ -6\ }x + \underline{\ 20\ })$

d. $x^2 + \underline{\ 8\ }x + 15 = (x + \underline{\ 5\ })(x + 3)$

b–c, e–f. To be completed by the student.

Lesson 2, Investigation 1, Applications Task 2 (p. 119)

a. Candidate A is top-rated by Tonya since A has the greatest row sum, and the row sum is the sum of the candidate's rating on all three issues.

b. Tonya's rating for A is $2(4) + 5 + 3(2) = 19$.

c–d. To be completed by the student.

Lesson 2, Investigation 1, Review Task 24 (p. 130)

a. $22x - 16$

b, d. To be completed by the student.

c. $14n^2 + 10n + 72$

Lesson 2, Investigation 1, Review Task 26 (p. 130)

a. 120 cm^3

b–c. To be completed by the student.

Lesson 2, Investigation 2, Applications Task 3 (p. 119)

The order of multiplication is crucial. The number of columns of the first matrix must be the same as the number of rows of the second matrix. Also in addition to that requirement, in order for the answer matrix to make sense in the context, the labels on the columns of the first matrix must match the labels on the rows of the second matrix.

a. The entries would represent total revenue and total profit for each of the first two weeks of Year 1.

b. **Year 1 Total**

	Revenue	Profit
Week 1	$17,256.60	$915.20
Week 2	$16,206.90	$859.00

c. **Year 2 Total**

	Revenue	Profit
Week 1	$17,386.70	$904.60
Week 2	$16,484.00	$857.10

d–e. To be completed by the student.

Lesson 2, Investigation 2, Reflections Task 13 (p. 125)

Only a square matrix can be multiplied by itself because it has the same number of columns as rows, so the number of columns in the first matrix is the same as the number of rows in the second matrix.

Lesson 2, Investigation 2, Review Task 32 (p. 131)

Students should supply specific examples as well as the solutions below.

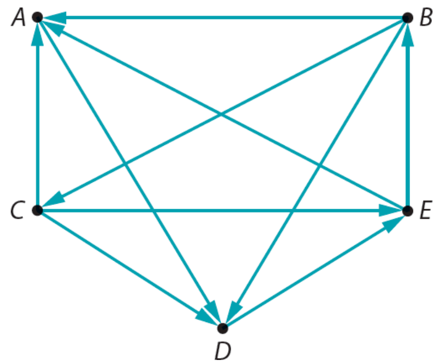
a. **i.** $x = 0$

ii. $x = 1$

b. To be completed by the student.

Lesson 2, Investigation 3, Applications Task 5 (p. 120)

a.



b.

	A	B	C	D	E
A	0	0	0	1	0
B	1	0	1	1	0
C	1	0	0	1	1
D	0	0	0	0	1
E	1	1	0	0	0

$= M$

c. Matrix M represents who beat whom, reading from row to column. When direct information is not available, ranking usually involves the logic that "better than" means that if you beat someone that beat someone, then you are better than the second person. Thus, with only the information that Anna beat Darien and Darien beat Emilio, we would expect that Anna would beat Emilio, and rank these three players Anna, Darien, Emilio. M^2 would represent these two-step expectations. M^3 would represent three-step expectations. Of course, the situation is never as clean as this. It is quite possible that there would be a two-step expectation that Anna would beat Emilio and simultaneously, because of other matches, that Emilio would beat Anna. Thus, these expectations are summed to create a final ranking.

The **row sums** of matrix M (above) indicate a tie in expectation between Bo and Chan.

	A	B	C	D	E
A	0	0	0	0	1
B	1	0	0	2	2
C	1	1	0	1	1
D	1	1	0	0	0
E	1	0	1	2	0

$= M^2$

Looking at the row sums of M^2 alone, a tie is still present, between Chan and Emilio.

	A	B	C	D	E
A	1	1	0	0	0
B	2	2	0	1	2
C	2	1	1	2	1
D	1	0	1	2	0
E	1	0	0	2	3

$= M^3$

Looking at the row sums of M^3 alone, a tie is still present.

	A	B	C	D	E
A	1	1	0	1	1
B	4	2	1	4	4
C	4	2	1	4	3
D	2	1	1	2	1
E	3	1	1	4	3

$= M + M^2 + M^3$

Examining the row sums of this matrix, the **highest row sum**, representing the highest expectations, for direct matches added to expectations for two and three stage comparisons, is for Bo (sum 15), then Chan, then Emilio, then Darien, then Anna. (Note that $M + M^2$ would also yield this ranking.)

Lesson 2, Investigation 3, Reflections Task 14 (p. 125)

a. Eventually, some power of A will be all zeroes. This is because there are not any circuits in the digraph, so all paths have finite length. At some point, the lengths of the paths corresponding to a power of the matrix A will be longer than the length of any path in the digraph, so all entries will be zero. In the food web, the longest paths are of length 4, so A^5 and all higher powers will contain all zeroes.

b. To be completed by the student.

c. Compute $A + A^2 + A^3 + A^4$. Compute up to the fourth power because 4 is the longest path length in the digraph. Thus, entries in $A + A^2 + A^3 + A^4$ show the number of paths of all possible lengths from one vertex to any other (see the matrix below). For each organism, look at the nonzero entries in the row corresponding to the organism. The organism corresponding to the column of the nonzero entry is farther up the food chain than the organism represented by the row.

	Bg	Fb	Fr	Gs	Mw	Sa	Sn	Sp	Yw
Bronze grackle	0	0	0	0	0	0	0	0	0
Flea beetle	1	0	2	2	0	0	0	1	2
Frog	0	0	0	1	0	0	0	0	0
Garter snake	0	0	0	0	0	0	0	0	0
Meadow willow	3	1	4	4	0	1	0	2	3
Sawfly	2	0	2	2	0	0	0	1	1
Snail	0	0	1	1	0	0	0	0	0
Spider	0	1	1	1	0	0	0	0	1
Yellow warbler	0	0	0	0	0	0	0	0	0

d–e. To be completed by the student.

Lesson 3, Investigation 1, Applications Task 2 (p. 145)

This task reviews once again if your student understands how to multiply matrices to set up a system of equations. When they multiply the matrices, they must multiply the rows of the first matrix times the columns of the second matrix. The answers are given below, but understanding where the answers came from is the important part.

a. $5a + 4c = 5$
$5b + 4d = 4$
$2a + 6c = 2$
$2b + 6d = 6$

b–c. To be completed by the student.

Lesson 3, Investigation 1, Connections Task 8 (p. 148)

a. $$\begin{bmatrix} 600 & 700 & 500 \end{bmatrix} \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} = \begin{bmatrix} 430 & 690 & 680 \end{bmatrix}$$

This model projects that 430 people will buy Nike shoes, 690 will buy Reebok shoes, and 680 will buy Fila shoes next year.

b–c. To be completed by the student.

Lesson 3, Investigation 1, Review Task 23 (p. 155)

a. $y = \frac{1}{3}x + 7$

b. $y = 3x + 7$

c–d. To be completed by the student.

Lesson 3, Investigation 2, Applications Task 4 (p. 146)

Hint: To begin this task let s represent the number of bricks in the short sides of the rectangle and the triangle, and let ℓ represent the number of bricks in the long sides. Then the equation for the rectangle in the system of equations is as follows: $2s + 2\ell = 50$. Use this example to help you write the other equation for the triangle and then solve the system.

Lesson 3, Investigation 2, Reflections Task 13 (p. 151)

a. Students should point out that sizes will not "fit" or that since matrix multiplication is not commutative, you cannot change the order.

b. Students should respond with reasoning similar to the following. Two general rules for solving equations are to "do the same thing to both sides" and "undo" something by "doing the opposite." Applying these rules to the equation $AX = D$, we see that in order to find X, we have to undo the multiplication by matrix A.

To do that, we multiply on the left by A^{-1} since $A^{-1} \bullet A = I$ and $I \bullet X = X$. Since we multiplied one side of the equation on the left by A^{-1}, we now have to do the same process to the other side of the equation. This yields $A^{-1} \bullet D$ (and not $D \bullet A^{-1}$). Some students may respond by noting that you cannot multiply on the right by A^{-1} since $A \bullet X \bullet A^{-1} \neq X$. Even in a situation where the sizes are compatible, $A \bullet X \bullet A^{-1}$ does not necessarily equal X since matrix multiplication is not commutative.

Lesson 3, Investigation 3, Connections Task 10 (p. 149)

a.

	Tourn	Std
Balls	6	1
Paddles	4	2

b. The matrix equation is $\begin{bmatrix} 6 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} 110 \\ 100 \end{bmatrix}$.

c–d. To be completed by the student.

Lesson 3, Investigation 3, Review Task 28 (p. 156)

a, b, d–f. To be completed by the student.

c.

$$\begin{aligned} 2x + 8(12 + 3x) &= 20x \\ 2x + 96 + 24x &= 20x \\ 96 + 2x + 24x &= 20x \\ 96 + 26x &= 20x \\ 96 + 26x - 26x &= 20x - 26x \\ 96 &= -6x \\ \frac{96}{-6} &= \frac{-6x}{-6} \\ -16 &= x \end{aligned}$$