Examples of Tasks from CCSS Edition Course 3, Unit 1

Getting Started
The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in English and Spanish, is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance. Comments in red type are not part of the solution.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Algebra and Functions page might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 1
Upon completion of this unit, students should be able to:

- recognize the differences between, as well as the complementary nature of, inductive and deductive reasoning.
- develop some facility in analyzing and producing deductive arguments in everyday contexts and in geometric, algebraic, and statistical situations.
- know and be able to use the relations among the angles formed when two lines intersect, including the special case of perpendicular lines.
- know and be able to use the necessary and sufficient conditions for two lines to be parallel.
- use symbolic notation to represent numerical patterns and relationships and use rules for transforming algebraic expressions and equations to prove those facts.
- distinguish between sample surveys, experiments, and observational studies and know the characteristics of a well-designed experiment.
- use statistical reasoning to decide whether one treatment causes a better result than a second treatment.

What Solutions are Available?

Lesson 1: Investigation 1—Review Task 29 (p. 27), Review Task 30 (p. 27)
Investigation 2—Applications Task 6 (p. 19), Applications Task 8 (p. 20), Reflections Task 20 (p. 24), Extensions Task 27 (p. 26), Review Task 33 (p. 28)

Lesson 2: Investigation 1—Applications Task 1 (p. 40), Review Task 30 (p. 50), Review Task 32 (p. 51)
Investigation 2—Applications Task 6 (p. 42), Reflections Task 17 (p. 46), Review Task 34 (p. 51), Review Task 35 (p. 51)

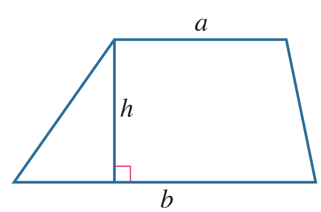
Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the CCSS Edition book.
For homework tasks in books with earlier copyright dates, see Helping with Homework.)

Lesson 1, Investigation 1, Review Task 29 (p. 27)

Here are 3 ways to find the area of a trapezoid:

- To find the area of a trapezoid you break it up into various polygons and sum the area of the polygons.
- If a and b are the lengths of the two parallel bases of a trapezoid, and h is its height, the area of the trapezoid A is $A = \frac{(a + b)}{2}(h)$.
- The formula simply stated: average the base lengths (parallel sides) and multiply by its height.



a. To be completed by the student. The height can be found from the grid.

b. To be completed by the student.

Hint: Use right triangle trigonometry or the relationship among side lengths of a 30˚-60˚ right triangle to find the height.

Lesson 1, Investigation 1, Review Task 30 (p. 27)

a. 30

b, d. To be completed by the student.

c. 18

Lesson 1, Investigation 2, Applications Task 6 (p. 19)

a. If a student is a sophomore at Calvin High School, then the student enrolls in physical education.

Hypothesis: A student is a sophomore at Calvin High School.
Conclusion: The student enrolls in physical education.

A statement written in if-then form is called a conditional statement. When writing statements in if-then form it is important not to change the meaning of the statement. Once you have it written in this form the "If" part is always the hypothesis and the "then" part is always the conclusion.

b. To be completed by the student.

c. Nothing can be concluded about Rosa. The fact is that *if* a student is a sophomore, *then* he or she enrolls in physical education; however, students in other classes may be able to enroll in physical education also.

When drawing conclusions you must pay close attention to the conditional statement you have as fact. Many students incorrectly answer this question by concluding *Rosa is a sophomore*. This is incorrect because other students, for example freshmen, may be able to enroll in physical education also. The error is interchanging the hypothesis and conclusion of the known fact.

Lesson 1, Investigation 2, Applications Task 8 (p. 20)

a. To be completed by the student.

b. **Statement I** A correct proof must contain no errors and use general reasoning rather than reasoning about a specific case. For example 6, 7, and 8 are three consecutive numbers that add up to 21. And 21 is divisible by 3. But this is *inductive reasoning* and not a deductive proof. It is an **example** (a specific case), which causes us to hypothesize the statement is correct and thus must be proved. A possible proof follows.

A possible proof for Statement I:
Let x, $x + 1$, $x + 2$ represent three consecutive positive integers. Since $x + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1)$, 3 is a factor of the sum. So, the sum is divisible by 3.

Lesson 1, Investigation 2, Reflections Task 20 (p. 24)

Inductive reasoning begins with examples, looks for a pattern or relationship, and then conjectures that the relationship holds for all similar situations. Deductive reasoning begins with known information and logically reasons to the truth of some statement. Inductive reasoning leads to plausibility (seems true), while deductive reasoning leads to certainty (a fact). Inductive reasoning is often used to develop or check conjectures that will be proved deductively.

Lesson 1, Investigation 2, Extensions Task 27 (p. 26)

A line reflection satisfies the first condition because each point in the plane is mapped to one point, either itself or to the endpoint for which the reflection line is the perpendicular bisector.

The second condition is to be completed by the student.

Lesson 1, Investigation 2, Review Task 33 (p. 28)

a. $10x + 35$

b. $2x^2 + 21x + 27$

c–f. To be completed by the student.

Lesson 2, Investigation 1, Applications Task 1 (p. 40)

a. $m\angle DBC = 90^\circ$

b. **i.** $m\angle 2 = 63^\circ$

ii–iii. To be completed by the student.

c. $m\angle ABG = 129^\circ$
$m\angle 1 = 39^\circ$

d. To be completed by the student.

Lesson 2, Investigation 1, Review Task 30 (p. 50)

a. 16 feet

b. 7.71 feet

c. 6.62 feet

d, e. To be completed by the student.

Lesson 2, Investigation 1, Review Task 32 (p. 51)

a, c. To be completed by the student.

b. *Hint:* You must find the probability that both marbles are red OR both marbles are blue. So, first find the probability of drawing a red on your first AND a red on your second draw. Then find the probability of drawing a blue on your first AND a blue on your second draw. Then add those probabilities together. The answer turns out to be $\frac{10}{16}$.

Lesson 2, Investigation 1, Review Task 33 (p. 51)

a. a^7

b. a^2

c–f. To be completed by the student.

Lesson 2, Investigation 2, Applications Task 6 (p. 42)

a. In order for the exiting light rays to be parallel, we must have $m\angle 2 + m\angle 5 = 180^\circ$. From the diagram, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ and $m\angle 4 + m\angle 5 + m\angle 6 = 180^\circ$. Combining these two equations, we get $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$. But since $m\angle 2 + m\angle 5 = 180^\circ$, this equation becomes $m\angle 1 + m\angle 3 + m\angle 4 + m\angle 6 = 180^\circ$. Since $m\angle 1 = m\angle 3$ and $m\angle 4 = m\angle 6$ (congruent angles from light reflection), $2m\angle 3 + 2m\angle 4 = 180^\circ$. Divide both sides of this equation by 2 to get $m\angle 3 + m\angle 4 = 90^\circ$. But in $\triangle XYQ$, $m\angle 3 + m\angle 4 + m\angle Q = 180^\circ$, so $m\angle Q = 90^\circ$.

b–d. To be completed by the student.

Lesson 2, Investigation 2, Reflections Task 17 (p. 46)

a. *Hint:* Two lines are perpendicular if and only if their slopes are opposite reciprocals of each other. For example, if one line has a slope of $\frac{3}{4}$, then the a perpendicular to it would have slope $-\frac{4}{3}$.

b–d. To be completed by the student.

Lesson 2, Investigation 2, Review Task 34 (p. 51)

a. M has coordinates (0, 5) and N has coordinates (3, 5).

b. The slopes of $\overline{MN}$ and $\overline{OB}$ are 0. So, the lines are parallel. (Students should show the slope calculations.)

c, d. To be completed by the student.

Lesson 2, Investigation 2, Review Task 35 (p. 51)

a. $x = \frac{21}{8}$

b. $x = 2$

c, d. To be completed by the student.

Lesson 3, Investigation 1, Applications Task 2 (p. 62)

a, b, d, e. To be completed by the student.

c. $(2m + 1)(2n) = 4mn + 2n$. To show this is even, we must write it as 2 times something. So, if we factor out a 2 from $4mn + 2n$, we get $2(2mn + n)$ and thus, by definition, an even number. Therefore, the product of an odd and even number is always an even number.

Lesson 3, Investigation 1, Applications Task 3 (p. 62)

a. $a(10) + b$ is an expression for the number.
$b(10) + a$ is an expression for the number with the digits reversed.
To show the sum of these two expressions will always be a multiple of 11, we must be able to write the sum in the form of 11 times some expression. So, add the original and new expressions together and factor: $[a(10) + b] + [b(10) + a] = 11a + 11b = 11(a + b)$

b. To be completed by the student.

Lesson 3, Investigation 1, Extensions Task 31 (p. 71)

Though Earth and (especially) Jupiter are not perfect spheres, we have assumed so in this problem that compares their sizes.

a. $333{,}000{,}000{,}000{,}000 = 3.33 \times 10^{14}$ and $\log (3.33 \times 10^{14}) = \log 3.33 + \log 10^{14} = \log 3.33 + 14 \approx 14.5224$. This reasoning uses the basic property of logs developed in Investigation 1.

b–e. To be completed by the student.

f. The formula for volume of a cube is $V = \frac{4\pi r^3}{3}$, so its volume is proportional to r^3. (The remainder of this part is to be completed by the student.)

Lesson 3, Investigation 1, Review Task 34 (p. 72)

a. $x = \frac{15}{19}$

b, c, e. To be completed by the student.

d. $x = 2$ or $x = -7$

f. $x = \pm\sqrt{5}$

Lesson 3, Investigation 1, Review Task 35 (p. 72)

a, b, e. To be completed by the student.

c. pq^5

d. p^2q

f. $\frac{q^5}{p^2}$

Lesson 3, Investigation 2, Applications Task 11 (p. 65)

a. (1) Distributive property
(2) Substitution using arithmetic

b. (1) Commutative Property of Addition
(2) Associative Property of Addition

c. To be completed by the student.

Lesson 3, Investigation 2, Connections Task 19 (p. 67)

a. **i.** $(3)^2 + 2(7)(3) + (7)^2 = 9 + 42 + 49 = 100 = 10^2 = (3 + 7)^2$

ii–iii. To be completed by the student.

b. $(x + p)^2 = (x + p)(x + p)$
$= (x + p)x + (x + p)p$

The remainder of the solution is to be completed by the student.

c. To be completed by the student.

Lesson 3, Investigation 2, Review Task 40 (p. 73)

a. $P(3) + P(4) + P(5) = \frac{1}{4}$

b. 3.16 rolls

c, d. To be completed by the student.

Lesson 4, Investigation 1, Applications Task 2 (p. 93)

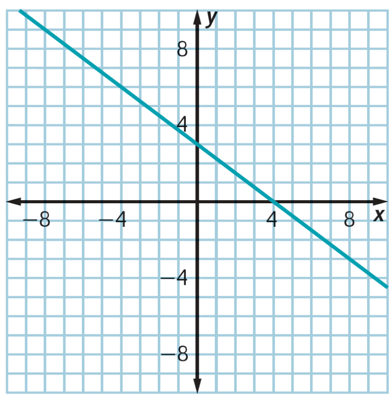
a. The larger the weight of the car, the lower the overall driver death rate.

b. $\frac{\text{467 deaths}}{\text{(4 million registered vehicles)(3 years)}} \approx 38.92$ deaths per million registered vehicle years

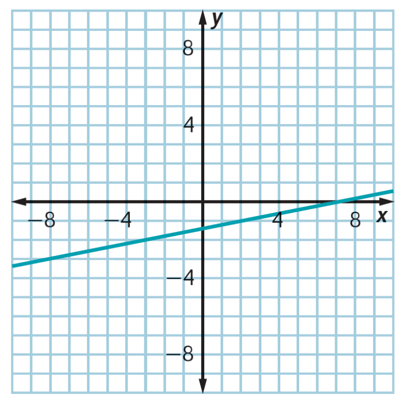
c, d. To be completed by the student.

Lesson 4, Investigation 1, Review Task 18 (p. 100)

a. $3x + 4y = 12$



b. $x - 5y = 7$



c–f. To be completed by the student.

Lesson 4, Investigation 2, Applications Task 3 (p. 93)

a–c, e. To be completed by the student.

d. Begin by making the assumption that the particular packaging of each steak made no difference in the log of the number of bacteria. Write the 6 measurements representing the 6 steaks on 6 slips of paper. Mix them thoroughly and deal out 3 to be "given" the plastic wrap. The remaining 3 steaks will be "given" the vacuum packaging. Because we are assuming that the treatment makes no difference in the response, the response for each steak will be the same as in the actual experiment. Compute the mean for each treatment group and subtract: *mean for commercial plastic wrap – mean for vacuum packaging*. Repeat this many times until you see the shape of the distribution. If the difference from the actual experiment is not in a tail of the distribution, there is no reason to abandon the assumption that the packaging made no difference. If the difference from the actual experiment is in a tail of the distribution, the assumption that the packaging made no difference is not plausible. So, you should conclude that the different packaging caused the difference in the means of the logs of bacteria count.

Lesson 4, Investigation 2, Applications Task 4 (p. 94)

These data are located in *CPMP-Tools* under Course 3, Statistics, Randomization Distribution, Unit 1 Reasoning and Proof, Mozart/Silence. You can then use the Randomization Distribution feature of your data analysis software to create an approximate distribution of the possible differences.

Lesson 4, Investigation 2, Connections Task 8 (p. 96)

a. Here is a partial list:

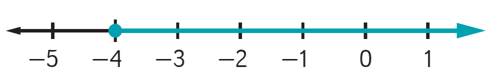
Fill in the missing two lines and then continue with Parts b, c, and d in your textbook.

Commercial Plastic Wrap	Mean of Plastic Wrap	Vacuum Wrap	Mean of Vacuum Wrap	Difference of Means
7.66, 6.98, 7.80	7.48	5.26, 5.44, 5.80	5.50	1.98
7.66, 6.98, 5.80	6.81	5.26, 5.44, 7.80	6.17	0.64
7.66, 6.98, 5.44	6.69	5.26, 5.80, 7.80	6.29	0.40
7.66, 6.98, 5.26	6.63	5.44, 5.80, 7.80	6.35	0.28
7.66, 7.80, 5.44	6.97	5.26, 5.80, 6.98	6.01	0.96
7.66, 7.80, 5.26	6.91	5.44, 5.80, 6.98	6.07	0.84
7.66, 5.26, 5.44	6.12	5.80, 6.98, 7.80	6.86	−0.74
7.66, 5.26, 5.80	6.24	5.44, 6.98, 7.80	6.74	−0.50
7.66, 5.26, 5.80	6.30	5.26, 6.98, 7.80	6.68	−0.38
7.66, 5.44, 5.80	5.50	7.66, 6.98, 7.80	7.48	−1.98
5.26, 5.44, 7.80	6.17	7.66, 6.98, 5.80	6.81	−0.64
5.44, 5.80, 7.80	6.29	7.66, 6.98, 5.44	6.69	−0.40
5.44, 5.80, 7.80	6.35	7.66, 6.98, 5.26	6.63	−0.28
5.26, 5.44, 6.98	5.89	7.66, 7.80, 5.80	7.09	−1.20
5.26, 5.80, 6.98	6.01	7.66, 7.80, 5.44	6.97	−0.96
5.44, 5.80, 6.98	6.07	7.66, 7.80, 5.26	6.91	−0.84
5.44, 6.98, 7.80	6.74	7.66, 5.26, 5.80	6.24	0.50
5.26, 6.98, 7.80	6.68	7.66, 5.44, 5.80	6.30	0.38

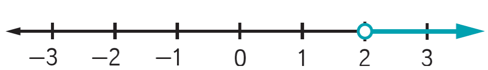
b–d. To be completed by the student.

Lesson 4, Investigation 2, Review Task 22 (p. 101)

a. $x \geq -4$



b. $x > 2$

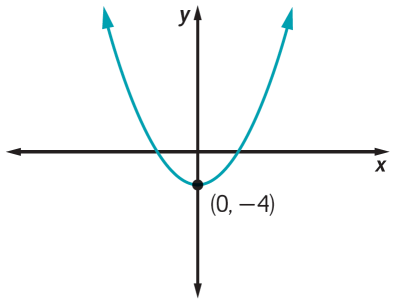


Note the difference between the “open” circle ○ and the “closed” (filled in) circle ● in Parts a and b. An “open” circle is placed on the graph to show that the number denoted at the circle is *not* included in the solution set, and a “closed” (filled in) circle is placed on the graph to show that the number denoted at the circle is included in the solution set.

c, d. To be completed by the student.

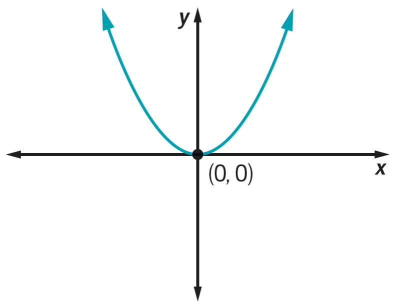
Lesson 4, Investigation 3, Review Task 24 (p. 101)

a.



Since the coefficient on x^2 is positive, the graph opens upward. The y-intercept is (0, −4). Since there is no linear term ($b = 0$), the y-axis is the symmetry line. Alternatively, students might recognize this function as a vertical shift of $y = x^2$. So, the graph has two x-intercepts, and the equation $f(x) = 0$ has two solutions.

b.



Since the coefficient on x^2 is positive, the graph opens upward. The vertex is (0, 0). So, the graph has one x-intercept, and the equation $f(x) = 0$ has one solution.

c–f. To be completed by the student.

Lesson 4, Investigation 3, Review Task 25 (p. 101)

a. $h(2) = 6 + 40(2) - 16(4) = 22$
Two seconds after the ball was released, it was 22 feet above the floor.

b. $h(0) = 6$; so, the basketball was released 6 feet above the floor.

c, d. To be completed by the student.

