Examples of Tasks from CCSS Edition Course 3, Unit 2

Getting Started

The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in English and Spanish, is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance. Comments in red type are not part of the solution.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See <u>Research on Communication</u>.

The <u>Algebra and Functions</u> page might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 2

Upon completion of this unit, students should be able to:

- write inequalities to express questions about functions of one or two variables.
- solve quadratic inequalities in one variable, and describe the solution set symbolically, as a number line graph, and using interval notation.

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- solve and graph the solution set of a linear inequality in two variables.
- solve and graph the solution set of a system of inequalities in two variables.
- solve linear programming problems involving two independent variables.

What Solutions are Available?

Lesson 1:	11: Investigation 1—Applications Task I (p. 118), Connections Task I2 (p. 12)						
	Reflections Task 19 (p. 123), Review Task 25 (p. 125)						
	Investigation 2—Applications Task 4 (p. 119), Applications Task 5 (p. 119),						
	Connections Task 15 (p. 122), Connections Task 16 (p. 123),						
	Review Task 30 (p. 125)						
	Investigation 3-Applications Task 6 (p. 119), Applications Task 8 (p. 120)						
Lesson 2:	Investigation 1—Applications Task 3 (p. 145), Connections Task 15 (p. 150),						
	Review Task 36 (p. 156)						
	Investigation 2—Applications Task 7 (p. 147), Review Task 39 (p. 156)						
	Investigation 3—Applications Task 10 (p. 148), Applications Task 14 (p. 149),						
	Reflections Task 23 (p. 152), Extensions Task 26 (p. 152)						

Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the CCSS Edition book. For homework tasks in books with earlier copyright dates, see <u>Helping with Homework</u>.)

Lesson 1, Investigation 1, Applications Task 1 (p. 118)

This task follows the same step-by-step procedure as Problem 1 in the investigation (page 110). The introductory paragraph on page 109 gives a sample equation and a nice picture of what the students need to find for each part of this task.

a. • $20 = -0.02x^2 + 1.3x + 1$

- The solutions can be found using the tables or graphs on your student's calculator, or using <u>CPMP-Tools</u>.
- The solutions are approximately x = 22.2 and x = 42.8.

b, **c**. To be completed by the student.

d. • $-0.02x^2 + 1.3x + 1 \ge 15$

- To be completed by the student.
- Since this equation has the \geq sign, the answer includes $\frac{-1.3 + \sqrt{1.3^2 4(-0.02)(-14)}}{2(-0.02)} \approx 13.6254$ and

 $\frac{-1.3 - \sqrt{1.3^2 - 4(-0.02)(-14)}}{2(-0.02)} \approx 51.3746$. To the nearest tenth-yard, solutions would be between 13.7 and

51.3. (It would be valuable to discuss with students why the lower endpoint is rounded up and the upper endpoint is rounded down.) The remainder of the solution is to be completed by the student.

Lesson 1, Investigation 1, Connections Task 12 (p. 121)

a. Recall: $\cos \theta = \frac{x}{r}$

The $\cos \theta$ is less than zero because the terminal side of the angle is in the second quadrant, and thus the *x*-coordinate is negative. The denominator of the cosine ratio is $\sqrt{x^2 + y^2}$ which is positive. Thus, $\cos \theta = \frac{x}{r}$ is negative in the second quadrant. Use similar reasoning to explain why $\sin \theta > 0$ and $\tan \theta < 0$.

b. To be completed by the student.

Lesson 1, Investigation 1, Reflections Task 19 (p. 123)

A function (with domain all real numbers) can only be greater than, less than, or equal to a certain value. Since f(x) > c when the value of x is between a and b (a < x < b), then f(x) must equal c when x = a and x = b. Also f(x) < c when x < a and x > b. So, $f(x) \le c$ when $x \le a$ or $x \ge b$.



Lesson 1, Investigation 1, Review Task 25 (p. 125)

- **a.** (1) Lead coefficient is positive, so the graph has a minimum point.
 - (2) $-\frac{b}{2a} = -2$, so the line of symmetry is x = -2.
 - (3) The constant term is -5, so the y-intercept is (0, -5).

b–d. To be completed by the student.

Lesson 1, Investigation 2, Applications Tasks 4-6 (p. 119)

Students should solve Applications Tasks 4–6 using the strategies they developed in Investigation 2. They should have examples from their class work.

- Sketch a graph showing the pattern of change you expect for the function involved.
- Use algebraic reasoning to locate key intersection points.
- Combine what you learn from your sketch and algebraic reasoning to solve the inequality.
- Record the solution using symbols and number line graphs.

Lesson 1, Investigation 2, Applications Task 4 (p. 119)

a. Use the strategies described on the previous page to solve $7t - t^2 < 0$.



 $7t - t^2 = 0$ Change the equation to equal to find the zeros. t(7 - t) = 0 Factor the nonzero side.

t = 0 or t = 7 Apply the Zero Product Property and solve each linear equation.

So, the *t*-coordinates of the intersection points are 0 or 7.

- To find where $7t t^2 < 0$, you need to look at the graph. Since it opens down, the function values are less than zero (below the *t*-axis) when t < 0 or t > 7.
- The solution represented on a number line graph is:

- **b.** To be completed by the student.
- **c–f.** To be completed by the student.

These inequality forms are solved with a different method of factoring. If the student needs assistance, a similar method is demonstrated in Applications Task 5 (p. 119) on the next page.

Lesson 1, Investigation 2, Applications Task 5 (p. 119)

a, **b**, **d**–**f**. To be completed by the student.

c. Use the strategies from page 3 to solve $5h^2 + 14h < 3$.



 $5h^2 + 14h = 3$ Set the expressions corresponding to the two functions $y = 5h^2 + 14h$ and y = 3 equal. Then find the points where they intersect.

 $5h^2 + 14h - 3 = 0$ Subtract 3 from both sides so that one side is zero. (5h-1)(h+3) = 0 $h = \frac{1}{5}$ or h = -3Factor the nonzero side.

Apply the Zero Product Property and solve each linear equation.

So, the *h*-coordinates of the intersection points are $\frac{1}{5}$ or -3.

We want to solve $5h^2 + 14h < 3$. Thinking of the expressions $5h^2 + 14h$ and 3 as functions, find the domain values (*h* values) for which the quadratic function is less than the linear function. From the graph, we see they intersect in two places. From the algebra, we know these two places are when $h = \frac{1}{5}$ or when h = -3. The quadratic function values are less than the line (below the line on the graph) for all h values between the two points of intersection.

• Thus the solution using symbols is $-3 < h < \frac{1}{5}$. The solution represented on a number line graph is:

-	+	+ +	+ +	+ +0	+ +	+ +	+ +	+ + + •
-4	-3	-2	—1	0	1	2	3	4

Lesson 1, Investigation 2, Connections Task 15 (p. 122)

- **a.** To be completed by the student.
- **b.** In Courses 1 and 2 of *Core-Plus Mathematics*, your student spent time developing an understanding of the different types of functions and their graphical representation. If they have their toolkit from last year, it would be very appropriate to ask them to find their entries from Course 2, Units 1 or 5, or Course 1, Unit 7. This would validate the importance of taking good notes during class and keeping them in an organized, usable state.

The first function $f(x) = \frac{50}{x^2}$ will always produce a positive value for any x, since the square of either a positive or a negative number is a positive number. The graph of the function colored red is above the x-axis, so this graph is f(x). Another fact confirming this choice is that the graph has reflection summetry across the y axis. This can be seen in the function rule since, for example, $f(x) = \frac{50}{x^2}$

symmetry across the *y*-axis. This can be seen in the function rule since, for example, $f(-3) = f(3) = \frac{50}{9}$. The function $g(x) = x^2 - 5x$ is called a quadratic and, since the function is not a fraction and has an x^2 in it, the graph will always be a parabola (U-shaped).

c. NOTE Make sure that the student colors the *x*-axis, not the graph itself, to show the values for which each statement is true.



Lesson 1, Investigation 2, Connections Task 16 (p. 123)

- **a.** Since both sides of this inequality contain $a^2 + b^2$, we are really comparing $-2ab \cos C$ and -2ab(-1). Since a factor of -2ab is in both of these expressions, we are really only comparing $\cos C$ and -1. Since we have a triangle, $m \angle C \neq 180^\circ$ and $m \angle C \neq 0^\circ$. So, $-1 < \cos C < 1$ or $\cos C > -1$ and $a^2 + b^2 - 2ab \cos C < a^2 + b^2 - 2ab(-1)$. (Recall that multiplying an inequality by a negative number switches the order of the inequality.)
- **b.** *Hint:* We know that $a^2 + b^2 2ab \cos C = c^2$ and we also know that $a^2 + 2ab + b^2 = (a + b)^2$. Use the statement in Part a and the preceding information to explain why it is true.
- c, d. To be completed by the student.

Lesson 1, Investigation 2, Review Task 30 (p. 125)

- **a.** Law of Sines; $AB \approx 5.47$
- **b.** Law of Cosines; $m \angle B \approx 54.0^{\circ}$

Lesson 1, Investigation 3, Applications Task 6 (p. 119)

a. Sketch the graphs of f(k) = k + 10 and $g(k) = \frac{24}{k}$ to look where the graph f(k) = k + 10 is greater than $g(k) = \frac{24}{k}$.



To solve the equation $k + 10 = \frac{24}{k}$, begin by multiplying both sides by the variable k to get $k^2 + 10k = 24$. The remainder of the solution is left to the student.

b–d. To be completed by the student.

Lesson 1, Investigation 3, Applications Task 8 (p. 120)

a–c. To be completed by the student.

d. Income from sale of the videos will exceed expenses for the videos as long as the club charges between \$58.39 and \$109.61. The greatest difference between income and expenses (or profit) occurs halfway between these two prices, at \$84. So, the club would make the most money by charging \$84 per video. However, if the club is interested in allowing the greatest number of members to purchase a video without losing money, rather than in making a profit, they should charge \$58.39 per video. It would make sense to charge any amount between \$58.39 and \$84.

Lesson 2, Investigation 1, Applications Task 3 (p. 145)

Students should have answers to similar problems in their class notes for Problem 4 on page 130.

- **a.** $7n_1 + 12n_2 \ge 11,000$
- **b.** *Hint:* To make the graph, determine the *x* and *y*-intercepts for $7n_1 + 12n_2 = 11,000$ and graph the lines containing the points. Then select a test point on one side of the line to find the region where $7n_1 + 12n_2 \ge 11,000$.

Lesson 2, Investigation 1, Connections Task 15 (p. 150)

a. *Hint:* Remember that if you have an equation of the form x = a or y = b, you are graphing a horizontal or vertical line. This is a square.

- **b.** *Hint:* To make the graph, determine the *x* and *y*-intercepts for any equation Ax + By = C and graph the lines containing the points. Then select a test point on one side of the line to find the region that satisfies the inequality.
- c, d. To be completed by the student.

Lesson 2, Investigation 1, Review Task 36 (p. 156)

- **a.** *Hint:* Remember that in order for a shape to be a rectangle, there must be a right angle. In order to have a right angle, the slopes of the adjacent lines must multiply to be -1.
- **b.** *Hint:* To facilitate finding the area of quadrilateral *ABCD*, students might consider a larger rectangle circumscribing *ABCD*.



c–e. To be completed by the student.

Lesson 2, Investigation 2, Applications Task 7 (p. 147)

- **a.** Plan 1: Feasible $2(40) + 1(20) = 100 \le 120$ liters of orange juice $2(40) + 3(20) = 140 \le 200$ liters of lemonade Plans 2-4 to be completed by the student.
- **b.** Plan 1: 9(40) + 12(20) = \$600
- Plans 2-4 to be completed by the student.
- **c.** To be completed by the student.

Lesson 2, Investigation 2, Review Task 39 (p. 156)

- **a.** $2\sqrt{21}$
- **b-d**, **g**, **h**. To be completed by the student.
- e. $\frac{5}{3}\sqrt{5}$
- f. $2a\sqrt{5}$

Lesson 2, Investigation 3, Applications Task 10 (p. 148)

- **a. i.** $2C + L \le 120$ liters of orange juice
 - ii–iii. To be completed by the student.

- **b.** To be completed by the student.
- **c.** The corners of the feasible region are (0, 66.67), (40, 40), and (60, 0). (*Hint:* You can use these points to be sure you have the other constraints correct and then use your objective function to maximize profit.)

Lesson 2, Investigation 3, Applications Task 14 (p. 149)

Choosing the correct variables for linear programming can be problematic for some students.

- **a.** The number of beginner and advanced classes offered are the two variables. Let *a* stand for the number of advanced classes and let *b* stand for the number of beginner classes. Let *P* stand for the number of children participating.
- **b.** *Hint:* There are four different constraints—two in the first bullet and one in each of the other two bullets.



c, e, f. To be completed by the student.

Lesson 2, Investigation 3, Reflections Task 23 (p. 152)

- a. Positive integer values make sense for most of the linear programming problems in this lesson.
- **b.** To be completed by the student.

Lesson 2, Investigation 3, Extensions Task 26 (p. 152)

- **a.** *Hint:* To help find this inequality, focus on points that will use up the whole plan. The ones to focus on are the ones that use up all the talk time or the standby time. For example, if you use up all the talk time minutes, you would get the equation $\frac{200}{200} + \frac{0}{7,200} = 1$; or if you used up all of your standby minutes, the equation would be $\frac{0}{200} + \frac{7,200}{7,200} = 1$.
- **b**, **c**. To be completed by the student.