

Examples of Tasks from CCSS Edition Course 3, Unit 6

Getting Started

The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in [English](#) and [Spanish](#), is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance. Comments in red type are not part of the solution.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See [Research on Communication](#).

The [Geometry and Trigonometry](#) page might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 6

Upon completion of this unit, students should be able to:

- state, prove, and apply various properties of a line tangent to a circle, central angles, chords, arcs, and radii of a circle.
- state, prove, and apply the Inscribed Angle Theorem and the property that inscribed angles that intercept the same or congruent arcs are congruent.
- analyze situations involving pulleys or sprockets to determine angular velocity and linear velocity.
- use sines and cosines to model aspects of circular motion and other periodic phenomena using both degrees and radians.

What Solutions are Available?

Lesson 1: Investigation 1—Applications Task 1 (p. 408), Connections Task 12 (p. 411), Review Task 32 (p. 418)

Investigation 2—Applications Task 4 (p. 408), Applications Task 6 (p. 409), Reflections Task 21 (p. 415), Extensions Task 27 (p. 416), Review Task 37 (p. 418)

Investigation 3—Applications Task 8 (p. 410), Applications Task 9 (p. 410), Connections Task 17 (p. 414), Review Task 40 (p. 419)

Lesson 2: Investigation 1—Applications Task 3 (p. 438), Applications Task 4 (p. 439), Extensions Task 27 (p. 448), Review Task 33 (p. 451)

Investigation 2—Applications Task 6 (p. 439), Connections Task 15 (p. 444), Review Task 39 (p. 453)

Investigation 3—Applications Task 8 (p. 440), Connections Task 18 (p. 445), Review Task 40 (p. 453)

Investigation 4—Applications Task 10 (p. 441), Applications Task 11 (p. 441), Connections Task 19 (p. 445), Extensions Task 30 (p. 449)

Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the CCSS Edition book.)

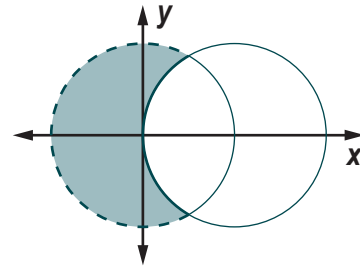
For homework tasks in books with earlier copyright dates, see [Helping with Homework](#).)

Lesson 1, Investigation 1, Applications Task 1 (p. 408)

- a. *Hint:* Draw lines that are perpendicular to the two edges of the square at points A and C . Which theorem learned in Investigation 1 justifies the location of the center?
- b. To be completed by the student.
- c. If a circle is placed inside (inscribed in) a square, the sides are tangent to the circle. (In Investigation 1, students learned that the tangent segments to a circle from an exterior point are congruent.)

Lesson 1, Investigation 1, Connections Task 12 (p. 411)

- a. **i, iii.** To be completed by the student.
 - ii. $x^2 + y^2 < r^2$
- b. **i.** $-3 < x < 6$ and $-3 < y < 3$, since each circle has radius 3.
 - ii. First, you need to be inside the far left circle (not on the circumference), so $x^2 + y^2 < 9$. However, the point must be outside of or on the right circle, so $(x - 3)^2 + y^2 \geq 9$ as well. A shaded diagram showing this region is at the right.
 - iii, iv. To be completed by the student.



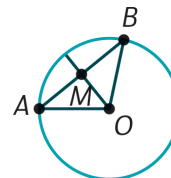
Lesson 1, Investigation 1, Review Task 32 (p. 418)

- a. $3x^3 + 13x^2 - 8x + 12$
- b–d. To be completed by the student.

Lesson 1, Investigation 2, Applications Task 4 (p. 408)

- a. **Proof Plan:** *If a line contains the center of a circle and the midpoint of a chord, then the line is perpendicular to the chord.* A picture of this conjecture is shown below.

- (1) Ask, “What information would allow me to conclude that $\overline{OM} \perp \overline{AB}$?”
- (2) Then ask, “What information would let me conclude the fact I need from Step 1?”



- (3) Finally, consider the facts from the hypothesis. “If a line contains the center of a circle and the midpoint of a chord, then the line is perpendicular to the chord.” Mark the given information on the diagram.

(4) Ask, “What already proven theorems use the given information to allow conclusions that lead to perpendicularity of \overline{OM} and \overline{AB} ?” (If you have not yet considered congruent triangles, this is a good option.)

(5) To be completed by the student.

b. To be completed by the student.

Lesson 1, Investigation 2, Applications Task 6 (p. 410)

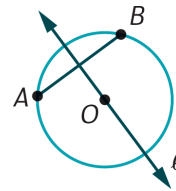
a. *Hint:* What did you learn in Investigation 2 about the location of the center of a circle? Consider using this theorem twice.

b. To be completed by the student.

Lesson 1, Investigation 2, Reflections Task 21 (p. 415)

Hint: There are two theorems that fit that criteria. They are listed below with a picture include.

(1) A line through the center of a circle bisects a chord if and only if the line is perpendicular to the chord. (This if-and-only-if statement could be written as two separate theorems.)

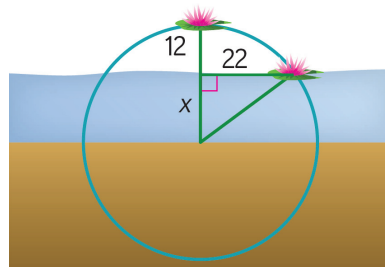


(2) The perpendicular bisector of a chord contains the center of a circle.

Students need to explain why these theorems follow from the fact that a circle is symmetric about any line through its center.

Lesson 1, Investigation 2, Extensions Task 27 (p. 416)

Hint: Examine the drawing at the right. Use the right triangle beneath the water and the fact that the radius of the circle can be expressed as $x + 12$ to find the length x . If solved correctly, $x \approx 14.2$ cm.



Lesson 1, Investigation 2, Review Task 37 (p. 418)

a. $3x^2 + \frac{15}{x^2}$

b. $\frac{64x^9}{y^6}$

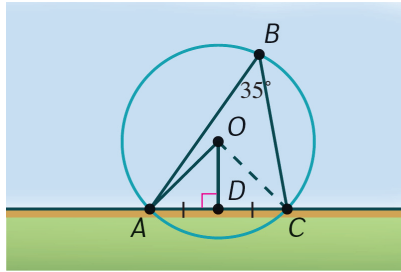
c, d. To be completed by the student.

Lesson 1, Investigation 3, Applications Task 8 (p. 410)

- a. *Hint:* Use Case 1 to help you prove Case 2. Since the diameter of the circle is \overline{BD} you can divide $\angle ABC$ into two inscribed angles and add them together.
- b. To be completed by the student.

Lesson 1, Investigation 3, Applications Task 9 (p. 410)

- a, c. To be completed by the student.
- b. To find the radius, first recall that a central angle is equal to its intercepted arc. Make an isosceles $\triangle AOC$ from the center of the circle O to help you find the radius, as the drawing below illustrates.



Since O is the center of the circle, let \overline{OD} be the perpendicular segment from O to chord \overline{AC} , so D is midpoint of \overline{AC} . Since the measure of the inscribed horizontal danger angle is 35° , the measure of $\widehat{AC} = 70^\circ$. In right triangle ADO , $m\angle AOD = 35^\circ$ and $AD = 0.3$ miles since D is the midpoint. Now use right triangle trigonometry to find the length of \overline{AO} .

Lesson 1, Investigation 3, Connections Task 17 (p. 414)

- a. *Hint:* Some of the coordinates are intuitive. To find the others, use the Pythagorean theorem and existing coordinates.
- b, d, e. To be completed by the student. (Do your answers for the coordinates of points D and E make sense?)
- c. *Hint:* How would knowing $m\angle DOC$ help you find the angle through which point A rotates to first reach the water at D ? How can you use trigonometric ratios to find that $m\angle DOC$ is approximately 36.9° ?

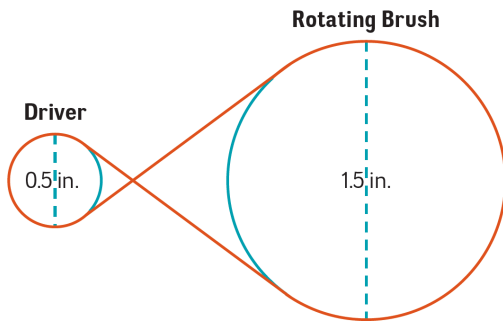
Lesson 1, Investigation 3, Review Task 40 (p. 419)

- a. $x = 16$
- b. $x = \pm 4\sqrt{2}$
- c–f. To be completed by the student.

Lesson 2, Investigation 1, Applications Task 3 (p. 438)

a, c, f. To be completed by the student.

b.



d. *Hint:* The point travels a distance equal to the circumference of the driver.

e. *Hint:* Find the transmission factor.

Lesson 2, Investigation 1, Applications Task 4 (p. 439)

a. *Hint:* The 10-cm pulley turns 1,500 rpm since it is attached to the crankshaft. Find the transmission factors from the crankshaft pulley to the compressor pulley and to the alternator pulley to find the respective angular velocities.

b. *Hint:* How far does the belt move each time the crankshaft goes around one time?

c. *Hint:* The linear velocity found in Part b was expressed in cm/min. Change the linear velocity to kilometers per minute. You should find that the linear velocity is π km/min. Show your work. Then find how long before the belt typically would show wear.

Lesson 2, Investigation 1, Extensions Task 27 (p. 448)

a. To be completed by the student.

b. \overline{AE} is one of the two common tangents. Let $CM = x$. Then $CN = 30 - x$. By similar triangles, $\frac{AM}{EN} = \frac{CM}{CN}$. Substituting, $\frac{8}{4} = \frac{x}{30 - x}$. The remainder of the solution is left for the student.

c. *Hint:* To find the length of the belt, you need to add four parts, the two common tangents, and the two portions of the circumference of the circle that the belt is touching. To find the portion of the circumference on each circle, find the measure of the central angle that does not have the belt touching it. Then use your answer to find the portion of the circumference the belt is touching.

Lesson 2, Investigation 1, Review Task 33 (p. 451)

a. Maximum point: $(5, 7)$
 x -intercepts: $(5 + \sqrt{7}, 0)$, $(5 - \sqrt{7}, 0)$
 y -intercept: $(0, -18)$

b. To be completed by the student.

Lesson 2, Investigation 2, Applications Task 6 (p. 439)

For this task, let the rotating transmitter/receiver be located at the origin on the (x, y) plane. Students should have reviewed the definitions of trigonometric functions on $0^\circ \leq \theta \leq 360^\circ$ while completing Review Task 32 on page 363. They should find these definitions in their Math Toolkit notes or on a copy of Selected Key Geometric Ideas from Courses 1 and 2. Students should be developing the habit of looking up ideas they may not remember.

- a. To find the x -coordinate, solve $x = 30 \cos 40^\circ$. To find the y -coordinate, solve $y = 30 \sin 40^\circ$.
- b–f. To be completed by the student.

Lesson 2, Investigation 2, Connections Task 15 (p. 444)

- a. To be completed by the student.
- b. *Hint:* Find the matrix $\begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}$ to determine the rotation matrix. To find the image points, multiply this matrix on the left of each of the column matrices in parts i–iii. To prove that this image point is correct geometrically, you could use trigonometry to show you have a 30° angle or recall the relationship that exists between the sides of a 30° - 60° right triangle to verify that you are correct.

Lesson 2, Investigation 2, Review Task 39 (p. 453)

- a, b, d. To be completed by the student.
- c. $\frac{x+5}{x-1}$

Lesson 2, Investigation 3, Applications Task 8 (p. 440)

- a. *Hint:* See the glossary for the definition of a radian. If still unclear, see pages 427–428 and investigation notes.
- b. To be completed by the student.
- c. Each radian intercepts an arc of length $r = 10$ cm, so the angular velocity is $80 \cdot 10 = 800$ cm/sec.
- d. *Hint:* In one second, how far will a point move at an angular velocity of v radians/sec?

Lesson 2, Investigation 3, Connections Task 18 (p. 445)

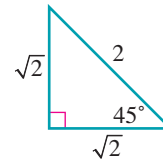
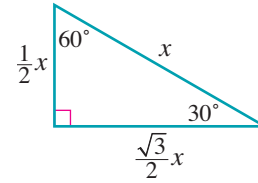
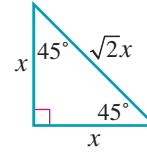
This task provides review of right triangle trigonometric ratios and 30° - 60° and 45° - 45° right triangle relationships (in terms of radians, $\frac{\pi}{6}$ - $\frac{\pi}{3}$ and $\frac{\pi}{4}$ - $\frac{\pi}{4}$ right triangles). In Lesson 1 Review Task 40 (page 419), students reviewed 30° - 60° and 45° - 45° right triangle relationships. These relationships along with the definition of trigonometric relationships for right triangles can be found on the student's copy of Selected Key Geometric Ideas from Courses 1 and 2. These ratios have been periodically used in Course 3. See Problem 5 (page 60) and Review Task 37 (page 73).

- a. Recall that $\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$. One way to represent the 30°-60° and 45°-45° right triangle relationships that align with the Key Geometric Ideas descriptions is shown in the first two diagrams at the right.

Another way to sketch a right triangle for the condition $\cos \frac{\pi}{4} = \cos 45^\circ = \frac{\sqrt{2}}{2}$ is shown in the third diagram at the right. So, $\sin \left(\frac{\pi}{4} \right) = \frac{\sqrt{2}}{2}$.

Here are the first three rows of the table to help check work.

t (in radians)	$\sin t$	$\cos t$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$



- b–e. To be completed by the student.

Lesson 2, Investigation 3, Review Task 40 (p. 453)

a.

NOW	NEXT
0	3
1	5.5
2	8
3	10.5
4	13
5	15.5

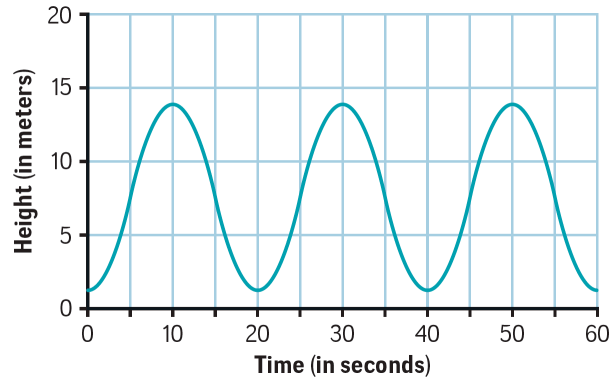
- b. To be completed by the student.

Lesson 2, Investigation 4, Applications Task 10 (p. 441)

- a. i, iii. To be completed by the student.
 ii. The amplitude is 3, the period is 20π , and the y -displacement is 5.
- b. To be completed by the student.

Lesson 2, Investigation 4, Applications Task 11 (p. 441)

a. The graph should look something like this:



b–d. To be completed by the student.

Lesson 2, Investigation 4, Connections Task 19 (p. 445)

a. The graph of $f(t) = -\cos t$ is a reflection of the graph of $g(t) = \cos t$ across the t -axis.

b, c. To be completed by the student.

Lesson 2, Investigation 4, Extensions Task 30 (p. 449)

a, c–f. To be completed by the student.

b. $y = 1 \cdot \sin (264 \text{ cycles/sec} \cdot 2\pi \text{ radians/cycle})x = \sin 528\pi x$, where x is in seconds.