Core-Plus Mathematics CCSS Edition
Unit and Lesson Objectives

Course 1

Unit 1: *Patterns of Change*

Unit Objectives

- Begin developing students' sensitivity to the rich variety of situations in which quantities vary in relation to each other
- Develop students' ability to represent relations among variables in several ways—using tables of numerical data, coordinate graphs, symbolic rules, and verbal descriptions—and to interpret data presented in any one of those forms
- Develop students' ability to recognize important patterns of change in single variables and related variables

Lesson Objectives

Lesson 1 *Cause and Effect*

- Develop disposition to look for cause-and effect relationships between variables
- Review and develop skills in organizing data in tables and graphs and using words to describe patterns of change shown in those representations
- Review or begin to develop knowledge about common patterns of change (linear, inverse, exponential, quadratic) and ability to use symbolic rules to represent and reason about those patterns
- Use tables, graphs, and rules to solve problems of cause-and-effect change

Lesson 2 *Change Over Time*

- Develop ability to recognize recursive patterns of change
- Develop ability to use calculators to iterate stages in a recursive pattern
- Develop ability to write *NOW-NEXT* rules to represent recursive patterns
- Develop ability to write and use spreadsheet formulas to explore recursive patterns of change (optional investigation)
- Use iteration to solve problems about population and money change over time

Lesson 3 *Tools for Studying Patterns of Change*

- Develop skill in writing rules that express problem conditions
- Review perimeter and area formulas for triangles, parallelograms, and circles, and the Pythagorean Theorem
- Develop skill in producing tables and graphs for functions
- Develop skill in using function tables, graphs, and computer algebra manipulations to solve problems that involve functional relationships, especially solving equations in one variable
- Develop informal knowledge about connections among function rules, tables, and graphs for linear, inverse, exponential, and quadratic relations

Unit 2: *Patterns in Data*

Unit Objectives

- Use various graphical displays of data to reveal important patterns in a data set and interpret those patterns in the context of the data
- Compute measures of center and variability for sets of data and interpret the meaning of those statistics
- Transform distributions by adding a constant or by multiplying by a positive constant and recognize how those transformations affect the shape, center, and spread of distributions

Lesson Objectives

Lesson 1 *Exploring Distributions*

- Construct dot plots, histograms, and relative frequency histograms
- Describe the shape of a distribution
- Compute and interpret the mean and median (from a list of values and from a frequency table)
- Estimate the mean and median from a histogram

Lesson 2 *Variability*

- Find and interpret percentiles and quartiles as measures of the position of a value in a distribution
- Find the five-number summary and the interquartile range (IQR) and interpret the IQR as a measure of variability
- Determine if a value is an outlier using a common rule
- Construct and interpret a box plot
- Compute and interpret deviations from the mean
- Compute or estimate and interpret the standard deviation as a measure of spread
- Predict the effect on the shape, center, and spread of a distribution when the same number is added to each value or when each value is multiplied by the same number

Unit 3: *Linear Functions*

Unit Objectives

- Recognize patterns in tables of sample values, in problem conditions, and in data plots that can be described by linear functions
- Write linear function rules to describe linear, or approximately linear, patterns in graphs or numerical data
- Use table, graph, or symbolic representations of linear functions to answer questions about the situations they represent: (1) Calculate y for a given x (i.e., evaluate functions); (2) Find x for a given y (i.e., solve equations and inequalities); and (3) Describe the rate at which y changes as x changes (i.e., determine slope)
- Rewrite linear expressions in equivalent forms

Lesson Objectives

Lesson 1 *Modeling Linear Relationships*

- Calculate the rate of change in one variable as another variable increases
- Describe the relationships among the graph, symbolic rule, table of values, and related situation for a linear function
- Interpret the meaning of the slope and y-intercept of the graph of a linear function in a context
- Write a rule for a linear function given its graph, two points, or a table of sample values
- Use linear functions to answer questions about the situations that they describe
- Estimate the graph and function rule for a line that fits a given set of data
- Use a linear model to predict the value of one variable given the value of the other and describe the rate of change in one variable as the other increases in a meaningful way
- Use a calculator or computer software to find the linear regression model for a set of data

Lesson 2 *Linear Equations and inequalities*

- Write linear equations and inequalities to express questions about linear functions
- Estimate solutions to linear equations and inequalities by inspecting appropriate graphs and tables of values and interpret the meaning of the solution in the real-world context
- Use "undoing" and "balancing" methods to solve simple linear equations and inequalities
- Use tables of values, graphs, and symbolic reasoning to solve systems of linear equations of the form $y = a + bx$ and $y = c + dx$

Lesson 3 *Equivalent Expressions*

- Write multiple expressions to represent a variable quantity from a real-world situation
- Use tables, graphs, and properties of numbers and operations to reason about the equivalence of expressions
- Rewrite linear expressions in equivalent forms by expanding, combining like terms, and factoring

Unit 4: *Discrete Mathematical Models*

Unit Objectives

- Understand and apply Euler paths and vertex coloring
- Use vertex-edge graphs to represent and solve problems related to paths, networks, and relationships among a finite number of objects
- Gain further experience in mathematical modeling by building and using vertex-edge graph models to solve problems in a variety of settings
- Develop skill in algorithmic problem solving: designing, using, and analyzing systematic procedures for solving problems
- Further develop skill in mathematical reasoning by exploring and reasoning about properties of vertex-edge graphs

Lesson Objectives

Lesson 1 *Euler Circuits: Finding the Best Path*

- Use vertex-edge graphs to model problems related to finding efficient routes—in this case, routes that use each edge exactly once
- Use Euler circuits, circuits through a graph that use each edge exactly once, to help solve such problems
- Learn and reason about properties of graphs and Euler circuits
- Investigate algorithms for constructing Euler circuits
- Use matrices to represent and analyze graphs

Lesson 2 *Vertex Coloring: Avoiding Conflict*

- Use vertex-edge graphs to model problems related to avoiding conflict in a variety of settings
- Color the vertices of a graph so that adjacent vertices have different colors
- Investigate algorithms for vertex coloring
- Use vertex coloring to solve a variety of problems, including assigning frequencies to radio stations, scheduling club meetings, and coloring the countries of a map

Unit 5: *Exponential Functions*

Unit Objectives

- Recognize and give examples of growth and decay situations in which exponential functions are likely to match the patterns of change that are observed or expected. This function-recognition skill should apply to information given in data tables, graphs, or verbal descriptions of related changing variables
- Develop ability to use reasoning, estimation, and curve-fitting utilities to find exponential functions to match patterns of change in exponential growth and decay situations. This should include rules in the "$y = \ldots$" and *NOW-NEXT* forms
- Use exponential rules to produce tables and graphs to answer questions about exponential change of variables
- Interpret an exponential function rule in order to sketch or predict the shape of its graph and the pattern of change in tables of values
- Describe major similarities and differences between linear and exponential patterns of change
- Develop skill in rewriting exponential and radical expressions in equivalent forms

Lesson Objectives

Lesson 1 *Exponential Growth*

- Develop disposition to look for and ability to recognize exponential growth patterns and phenomena
- Develop ability to represent exponential functions with rules in the form $y = a(b^x)$ where $a > 0$ and $b > 1$
- Develop ability to write *NOW-NEXT* rules for exponential growth patterns
- Develop ability to use tables and graphs to solve problems about exponential growth
- Develop ability to use reasoning, estimation, and curve-fitting utilities to model data patterns exhibiting exponential-type trends
- Develop skill in use of standard rules for writing exponential expressions in equivalent forms

Lesson 2 *Exponential Decay*

- Develop ability to recognize patterns of change characterizing exponential decay phenomena
- Develop ability to write explicit $y = a(b^x)$ and *NOW-NEXT* rules for exponential decay functions
- Develop ability to interpret zero and fractional exponents and to calculate or estimate values of expressions with those exponents
- Develop ability to interpret half-life of decay phenomena and to use symbolic rules, tables, and graphs to estimate those values
- Develop ability to use reasoning, estimation, and curve-fitting utilities to model exponential decay patterns
- Use symbolic rules, tables, and graphs to solve problems involving exponential decay
- Develop skill in use of standard rules for writing exponential expressions in equivalent forms
- Develop skill in simplifying radicals

Unit 6: *Patterns in Shape*

Unit Objectives

- Recognize and classify common two- and three-dimensional shapes
- Visualize and represent two- and three-dimensional shapes
- Analyze and apply properties of polygons and polyhedra
- Use rigid transformations to verify SSS, SAS, ASA conditions for congruence of triangles and use these conditions in solving problems
- Begin to develop ability to establish properties of shapes by careful reasoning from definitions and given or assumed facts

Lesson Objectives

Lesson 1 *Triangles, Quadrilaterals, and Their Properties*

- Discover and apply the Triangle Inequality and its analog for quadrilaterals
- Investigate rigidity of two-dimensional shapes
- Discover and apply properties of quadrilateral linkages, including those with rotating bars
- Discover and verify using rigid transformations (translation, rotation about a point, and line reflection) combinations of side and angle conditions that are sufficient for testing the congruence of two triangles: Side-Side-Side (SSS), Side-Angle-Side (SAS), Angle-Side-Angle (ASA) Use congruence conditions to reason about properties of isosceles triangles and select properties of parallelograms
- Use congruence conditions to reason about properties of isosceles triangles and select properties of parallelograms
- Use area and congruence relationships to justify why the Pythagorean Theorem and its converse are true, and use these results to solve problems involving right triangles
- Recall, justify derivations of, and use formulas to find areas of triangles and special quadrilaterals

Lesson 2 *Polygons and Their Properties*

- Discover and apply properties of the interior, exterior, and central angles of polygons
- Recognize and describe line and rotational symmetries of polygons and other two-dimensional shapes
- (Re)discover which triangles, quadrilaterals, and regular polygons will tile a plane and explore semiregular tessellations
- Recognize and describe symmetries of tessellations, including translation symmetry

Unit 7: *Quadratic Functions*

Unit Objectives

- Recognize patterns in tables of sample values, in problem conditions, and in data plots that can be described by quadratic functions
- Write quadratic function rules to describe quadratic, or approximately quadratic, patterns in graphs or numerical data
- Use table, graph, or symbolic representations of quadratic functions to answer questions about the situations they represent: (1) Calculate y for a given x (i.e., evaluate functions); (2) Find x for a given y (i.e., solve equations and inequalities); and (3) Describe the rate at which y changes as x changes
- Rewrite simple quadratic expressions in equivalent forms by expanding or factoring given expressions and/or by combining like terms

Lesson Objectives

Lesson 1 *Quadratic Patterns*

- Determine patterns of change associated with quadratic functions
- Use tables of values and graphs to estimate answers for questions about situations modeled by quadratic functions
- Describe the effects of each parameter in the function rule $y = ax^2 + bx + c$

Lesson 2 *Equivalent Quadratic Expressions*

- Find symbolic rules for quadratic functions using data modeling and reasoning
- Determine whether two given quadratic expressions are equivalent
- Decide on most useful equivalent forms of quadratics for different question types
- Create equivalent quadratic expressions by expanding products of linear factors
- Factor quadratic expressions by extracting common linear factors

Lesson 3 *Solving Quadratic Equations*

- Write quadratic equations and inequalities to express questions about quadratic functions
- Find exact values of solutions for quadratic equations in the form $ax^2 + c = d$ and $ax^2 + bx = 0$ by reasoning and factoring
- Relate factored forms of quadratic expressions to x-intercepts of graphs for the related functions
- Solve quadratic equations by using the quadratic formula
- Describe the possible number of real solutions for quadratic equations and illustrate the possibilities with graphs

Unit 8: *Patterns in Chance*

Unit Objectives

- Construct sample spaces and probability distributions and use them to understand chance situations involving equally likely outcomes
- Use the Addition Rule and its special case for mutually exclusive events to compute $P(A \text{ and } B)$
- Design and carry out simulations to decide whether the probability model is consistent with the data
- Use the Law of Large Numbers to understand situations involving chance
- Use geometric diagrams to solve probability problems that involve continuous variables

Lesson Objectives

Lesson 1 *Calculating Probabilities*

- Construct sample spaces for chance situations involving equally likely outcomes
- Construct probability distributions from sample spaces
- Identify mutually exclusive (disjoint) events
- Compute $P(A \text{ and } B)$ using the Addition Rule or its special case for mutually exclusive events

Lesson 2 *Modeling Chance Situations*

- Design and carry out simulations to decide whether the probability model is consistent with the data
- Use the Law of Large Numbers to understand situations involving chance
- Use tables of random digits to perform simulations and understand some properties of random digits
- Use random numbers to perform simulations in situations that involve continuous variables
- Use geometric diagrams to solve probability problems that involve continuous variables

Course 2

Unit 1: *Functions, Equations, and Systems*

Unit Objectives

- Review familiar families of single variable functions (especially linear, exponential, and quadratic functions)
- Recognize direct and inverse variation functions with one or more independent variables, express those relationships in symbolic form, and manipulate those expressions into equivalent useful forms
- Recognize and represent graphically and symbolically relationships in which one variable is a linear function of two independent variables and graph solutions of equations in the form "$ax + by = c$"
- Set up and solve systems involving two linear equations with two variables by use of graphing, substitution, and elimination methods. Recognize whether systems have 0, 1, or 2 solutions by inspecting the equations

Lesson Objectives

Lesson 1 *Direct and Inverse Variation*

- Recognize numeric and graphic patterns of change in direct and inverse variation relationships
- Express direct and inverse variation relationships in symbolic forms
- Recognize and represent relationships between variables that can be modeled by power functions $y = ax^r$ $(r \neq 0)$
- Solve problems involving direct and inverse variation

Lesson 2 *Multivariable Functions*

- Write rules to define functions of two variables that combine direct and inverse variation
- Solve for one variable in terms of the others in situations where the variables are related by direct and inverse variation
- Write equations in the general form $ax + by = c$ to express conditions relating two variables
- Solve linear equations for one variable in terms of the other
- Graph linear equations in the form $ax + by = c$

Lesson 3 *Systems of Linear Equations*

- Write systems of linear equations to match given problem conditions
- Solve linear systems by graphing, substitution, and elimination methods
- Recognize linear systems with zero or infinitely many solutions by inspecting graphs, equation forms, and results of reasoning by substitution and elimination

Unit 2: *Matrix Methods*

Unit Objectives

- See the interconnectedness of mathematics through use of matrices to solve problems in algebra, geometry, statistics, and discrete mathematics
- Use matrices to organize, display, and analyze data from a variety of contexts, such as archeology, sociology, ecology, sports, and business
- Understand, carry out, and interpret matrix operations—row and column sums, matrix addition and subtraction, scalar multiplication (multiply a matrix by a number), and matrix multiplication
- Understand and apply properties of matrices and matrix operations, compare properties of matrices to those of real numbers, and thereby gain a gentle introduction to algebraic structure
- Use matrices to solve systems of two linear equations
- Compare and analyze different methods for solving systems of two linear equations by considering limitations, advantages, and disadvantages of methods learned in this and prior units

Lesson Objectives

Lesson 1 *Constructing, Interpreting, and Operating On Matrices*

- Construct matrices to organize, display, and analyze information
- Interpret given matrices
- Understand, carry out, and interpret matrix operations—row and column sums, matrix addition and subtraction, and scalar multiplication

Lesson 2 *Multiplying Matrices*

- Understand, carry out, and interpret matrix multiplication
- Use matrix multiplication, including powers of matrices, to solve problems in a variety of settings
- Represent a vertex-edge graph as a matrix and use powers of that matrix to analyze the situation modeled by the vertex-edge graph

Lesson 3 *Matrices and Systems of Linear Equations*

- Examine properties of operations with matrices
- Compare properties of matrices with those of real numbers
- Use matrices and their properties to solve systems of linear equations
- Review, analyze, and compare various methods for solving systems of linear equations

Unit 3: *Coordinate Methods*

Unit Objectives

- Use coordinates to represent points, lines, and geometric figures in a plane and on a computer or calculator screen
- Use coordinate representations of figures to analyze and reason about their properties
- Use coordinate methods and programming techniques as a tool to implement computational algorithms, to model rigid transformations and similarity transformations, and to investigate properties of shapes that are preserved under various transformations
- Build and use matrix representations of polygons and transformations and use these representations to create computer animations

Lesson Objectives

Lesson 1 *A Coordinate Model of a Plane*

- Use coordinates to represent points, lines, and geometric figures in a plane
- Develop and use coordinate representations of geometric ideas such as distance, slope, and midpoint to analyze properties of lines and shapes
- Design algorithms for programming calculators or computers to perform routine geometry-related computations
- Develop and use equations for circles in a coordinate plane
- Reason with general coordinates to establish properties of triangles, quadrilaterals, and circles

Lesson 2 *Coordinate Models of Transformations*

- Use coordinates to develop function rules modeling translations, line reflections, and rotations and size transformations centered at the origin
- Use coordinates to investigate properties of figures under one or more rigid transformations or under similarity transformations
- Explore the concept of function composition using successive application of two transformations

Lesson 3 *Transformations, Matrices, and Animation*

- Use coordinate rules for rotations about the origin to develop corresponding matrix representations
- Use coordinate rules for size transformations centered at the origin to develop corresponding matrix representations
- Use matrix representations of shapes and transformations to create simple animations involving rotations and size transformations

Unit 4: *Regression and Correlation*

Unit Objectives

- Describe the shape of a cloud of points on a scatterplot and describe the association between the two variables
- Interpret the coefficients of the regression equation, learn some properties of the regression line, and understand that a regression line is an appropriate way to summarize the bivariate relationship only if the points form an elliptical cloud
- Compute and interpret Pearson's correlation and understand that a strong correlation does not imply that one variable causes the other
- Determine whether a point is influential on the correlation and on the equation of the least squares regression line

Lesson Objectives

Lesson 1 *Bivariate Relationships*

- Construct scatterplots with appropriate labels and scales
- Describe shapes of clouds of points on scatterplots (linear, curved, vary in strength)
- Identify types of association (positive and negative, strong and weak, perfect and none, linear and nonlinear)
- Identify clusters and different types of outliers
- Read and interpret a scatterplot matrix

Lesson 2 *Least Squares Regression and Correlation*

- Understand that a linear model is appropriate when the points form an elliptical cloud
- Compute errors in prediction and residuals and locate residuals on the plot
- Understand that for the regression equation, the sum of the residuals is 0
- Understand the regression line as the line that minimizes the sum of the squared residuals
- Verify that the point $(\overline{x}, \overline{y})$ is on the regression line
- Determine whether an outlier is an influential point
- Compute and interpret Pearson's r as a measure of how closely the points cluster about the regression line
- Interpret Pearson's r as the rescaled sum of the products $(x - \overline{x}, y - \overline{y})$
- Learn that the size of the correlation is not an indication of whether a linear model is appropriate
- Know that adding a constant to each value or multiplying by a positive constant does not change the correlation
- Understand that association does not mean that one of the variables causes the other
- Identify possible explanations (cause-and-effect, lurking variable) for an association and illustrate with a directed graph
- Identify the explanatory variable and the response variable

Unit 5: *Nonlinear Functions and Equations*

Unit Objectives

- Generalize the definition of function and introduce "$f(x)$" notation for functions and the concepts of domain and range
- Construct rules for quadratic functions based on given properties such as x-intercepts, y-intercept, and maximum/minimum point
- Write quadratic expressions in equivalent expanded or factored form
- Solve quadratic equations by factoring, by applying the quadratic formula, and by a CAS
- Write and solve equations that represent questions about "real-life" situations involving comparison of a linear function and either an inverse variation or quadratic function
- Estimate solutions to equations in the form $ax + b = \frac{k}{x}$ by using tables or graphs and solve those kinds of equations algebraically
- Estimate solutions to equations in the form $mx + d = ax^2 + bx + c$ using tables or graphs and solve those equations algebraically
- Use common logarithms to "linearize" exponential patterns of growth that occur in measurement of sound intensity, acidity (or alkalinity) of liquids, and earthquake intensity
- Use logarithms to solve exponential equations

Lesson Objectives

Lesson 1 *Quadratic Functions, Expressions, and Equations*

- Distinguish relationships between variables that are functions from those that are not
- Use $f(x)$ notation to represent functions and the common questions about functions that arise in applied problems
- Identify domain and range of functions
- Construct rules for quadratic functions based on given properties such as x-intercepts, y-intercept, and maximum/minimum point
- Write quadratic expressions in equivalent expanded or factored form
- Solve quadratic equations by factoring, by applying the quadratic formula, or by a CAS

Lesson 2 *Nonlinear Systems of Equations*

- Write an equation or inequality to represent a question about a "real-life" situation involving a comparison between a linear function and either an inverse variation or quadratic function
- Estimate solutions to equations in the form $ax + b = \frac{k}{x}$ using tables or graphs and solve algebraically
- Estimate solutions to equations in the form $mx + d = ax^2 + bx + c$ using tables or graphs and solve algebraically

Lesson 3 *Common Logarithms and Exponential Equations*

- Recognize what is meant by "taking the common logarithm" of a real number
- Be able to rewrite any real number as a power of 10 by finding common logarithms
- Use common logarithms to solve exponential equations, both in and out of context

Unit 6: *Modeling and Optimization*

Unit Objectives

- Understand and apply minimum spanning trees, Hamilton circuits, the Traveling Salesperson Problem, and critical paths (including ideas from the Critical Path Method, CPM, which is also called the Program Evaluation and Review Technique, PERT)
- Further develop skill in mathematical modeling by modeling and solving problems with vertex-edge graphs
- Further develop skill in algorithmic problem solving by designing, using, and analyzing systematic procedures for solving problems involving vertex-edge graphs
- Further develop the ability to recognize, formulate, and solve optimization problems, particularly network optimization problems

Lesson Objectives

Lesson 1 *Network Optimization*

- Understand and apply minimum spanning trees, Hamilton circuits, and the Traveling Salesperson Problem (TSP)
- Compare and contrast graph topics: TSP vs. minimum spanning trees, Hamilton vs. Euler circuits, matrices and graphs
- Further develop skill in mathematical modeling, particularly modeling with vertex-edge graphs
- Further develop skill in algorithmic problem solving, particularly designing, using, and analyzing algorithms for minimum spanning trees and the TSP
- Further develop the ability to recognize, formulate, and solve optimization problems, particularly related to optimum spanning networks

Lesson 2 *Scheduling Projects using Critical Paths*

- Construct and interpret a project digraph
- Determine earliest finish time for a project consisting of many tasks
- Understand and apply critical paths and critical tasks in the context of project scheduling
- Further develop skill in mathematical modeling, algorithmic problem solving, and optimization

Unit 7: *Trigonometric Methods*

Unit Objectives

- Explore the sine, cosine, and tangent functions defined in terms of a point on the terminal side of an angle in standard position in a coordinate plane
- Explore properties of the sine, cosine, and tangent ratios of acute angles in right triangles and use those ratios to solve applied problems
- Derive the Law of Sines and the Law of Cosines and use those laws to determine measures of sides and angles for non-right triangles
- Use the Law of Sines and Law of Cosines to solve a variety of applied problems that involve triangulation
- Describe the conditions under which two, one, or no triangles are determined given the lengths of two sides and the measure of an angle not included between the two sides

Lesson Objectives

Lesson 1 *Trigonometric Functions*

- Determine values of the sine, cosine, and tangent functions of an angle in standard position in a coordinate plane
- Determine the sine, cosine, and tangent of an acute angle in a right triangle, and determine the angle given one of those ratios
- Solve problems involving indirect measurement that can be modeled using right triangles
- Explore basic properties of the sine, cosine, and tangent functions with reference to their interrelationships and their patterns of change as the angle measure changes

Lesson 2 *Using Trigonometry in Any Triangle*

- Derive the Law of Sines and the Law of Cosines
- Determine measures of sides and angles of triangles using the Law of Sines and Law of Cosines
- Use these laws to solve problems involving indirect measurement and analysis of mechanisms that use triangles with a side of variable length
- Determine whether two, one, or no triangles are possible when the lengths of two sides and the measure of an angle not included between these sides are known

Unit 8: *Probability Distributions*

Unit Objectives

- Interpret and compute conditional probabilities
- Use the Multiplication Rule to find $P(A \text{ and } B)$, when events A and B are independent and when they are not independent
- Compute the expected value (mean) of a probability distribution
- Identify waiting-time situations and construct waiting-time distributions

Lesson Objectives

Lesson 1 *The Multiplication Rule*

- Use an area model to find the probability that two independent events both occur
- Compute conditional probabilities
- Determine if two events are independent
- Use the Multiplication Rule to find the probability that two events both occur when the events are or are not independent

Lesson 2 *Expected Value*

- Compute the fair price (expected value) of insurance and games of chance
- Develop and use a formula for the expected value of a probability distribution

Lesson 3 *The Waiting-Time Distribution*

- Use simulation to construct an approximate waiting-time distribution and understand why the shape is skewed to the right
- Recognize rare events in a waiting-time situation
- Develop and use the formula to construct the probability distribution for a waiting-time situation
- Discover the formula for the expected value of a waiting-time distribution
- Understand that some infinite series have a finite sum

Course 3

Unit 1: *Reasoning and Proof*

Unit Objectives

- Recognize the differences between, as well as the complementary nature of, inductive and deductive reasoning
- Develop some facility in analyzing and producing deductive arguments in everyday contexts and in geometric, algebraic, and statistical contexts
- Know and be able to use the relations among the angles formed when two lines intersect, including the special case of perpendicular lines
- Know and be able to use the necessary and sufficient conditions for two lines to be parallel
- Use symbolic notation to represent numerical patterns and relationships and use rules for transforming algebraic expressions and equations to prove those facts
- Distinguish between sample surveys, experiments, and observational studies; know the characteristics of a well-designed experiment
- Use statistical reasoning to decide whether one treatment causes a better result than a second treatment

Lesson Objectives

Lesson 1 *Reasoning Strategies*

- Recognize the role of inductive reasoning in making conjectures and recognize the limitations of inductive reasoning
- Recognize the need for proof and be able to create a simple deductive argument to prove a mathematical assertion
- Create a counterexample to prove a claim is false
- Write if-then statements and their converses and use if-then reasoning patterns in arguments
- Use inductive reasoning to develop line reflection assumptions and use deductive reasoning to justify line reflection properties

Lesson 2 *Geometric Reasoning and Proof*

- Know and be able to use the angle relationship theorems involving two intersecting lines
- Know and be able to use the theorems justifying the construction of a line perpendicular to a given line through a given point and the construction of a line parallel to a given line through a given point
- Know and be able to use the angle relationship theorems involving two parallel lines cut by a transversal and their converses
- Know and be able to use the angle sum theorem and the exterior angle theorem for triangles

Lesson 3 *Algebraic Reasoning and Proof*

- Use algebraic notation—letters, expressions, equations, and inequalities—to represent general patterns and relationships among variables
- Use algebraic transformations of expressions, equations, and inequalities to establish general propositions about quantitative relationships

Lesson 4 ***Statistical Reasoning***

- Know the characteristics of a well-designed experiment
- Understand the placebo effect
- Under the hypothesis of no treatment effect, construct an approximate sampling distribution for the difference of two means by rerandomizing
- Use a randomization test to decide if an experiment provides statistically significant evidence that one treatment is more effective than another
- Distinguish between three types of statistical studies—sample surveys, experiments, and observational studies—and understand what inference can be made from each

Unit 2: *Inequalities and Linear Programming*

Unit Objectives

- Write inequalities to express questions about functions of one or two variables
- Solve quadratic inequalities in one variable, and describe the solution set symbolically, as a number line graph, and using interval notation
- Solve and graph the solution set of a linear inequality in two variables
- Solve and graph the solution set of a system of inequalities in two variables
- Solve linear programming problems involving two independent variables

Lesson Objectives

Lesson 1 *Inequalities in One Variable*

- Write inequalities to express questions about functions of one or two variables
- Given a graph of one or more functions, solve inequalities related to the function(s)
- Solve quadratic inequalities in one variable by solving the corresponding equation algebraically and reasoning about the graph of the related function(s)
- Describe the solution set of an inequality in one variable symbolically, as a graph on a number line, and using interval notation

Lesson 2 *Inequalities with Two Variables*

- Graph the solution set of a linear inequality in two variables
- Graph the solution set of a system of inequalities in two variables
- Solve linear programming problems involving two independent variables

Unit 3: *Similarity and Congruence*

Unit Objectives

- Build skill in using inductive and deductive reasoning to first discover and then prove geometric relationships and properties based on similarity and congruence of triangles
- Develop facility in producing deductive arguments in geometric situations using synthetic, coordinate, and transformational methods
- Know and be able to use triangle similarity and congruence theorems
- Know and be able to use properties of special centers of triangles
- Know and be able to use the necessary and sufficient conditions for quadrilaterals to be (special) parallelograms and for special quadrilaterals to be congruent
- Know and be able to use properties of size transformations and rigid transformations (line reflections, translations, and rotations) to prove sufficient conditions for congruence of triangles and solve problems

Lesson Objectives

Lesson 1 *Reasoning about Similar Figures*

- Identify similar polygons and determine the scale factor of similar polygons
- Review and extend understanding of the Laws of Sines and Cosines
- Know and be able to use the three theorems providing sufficient conditions to prove triangles are similar (SSS, SAS, AA)
- Continue to develop the ability to write both synthetic and coordinate arguments
- Discover and prove properties of size transformations (dilations) using a synthetic approach

Lesson 2 *Reasoning about Congruent Figures*

- Understand congruence of figures as a special case of similarity of figures
- Know and be able to use the four theorems providing sufficient conditions to prove triangles are congruent (SSS, SAS, AAS, ASA)
- Know and be able to use properties of the incenter, circumcenter, and centroid of a triangle
- Continue to develop the ability to write both synthetic and coordinate arguments
- Know and be able to use both necessary and sufficient conditions for quadrilaterals to be (special) parallelograms
- Know and be able to use the Midpoint Connector Theorems for Triangles and Quadrilaterals
- Discover and prove properties of rigid transformations using a synthetic approach
- Know and use the key ideas of rigid transformations in proving sufficient conditions for congruence of triangles and composites of a size transformation and rigid transformations in proving sufficient conditions for similarity of triangles

Unit 4: *Samples and Variation*

Unit Objectives

- Understand the standard deviation as a measure of variability and the normal distribution as a model of variability
- When appropriate, fit a normal distribution to a set of data and use it to estimate percentages
- Construct binomial probability distributions and find and interpret expected values
- Use a random sample to make an inference about whether a specified proportion p is plausible as the population parameter
- Use concepts of probability to create strategies for product testing
- Understand the Central Limit Theorem and how it is applied to product testing

Lesson Objectives

Lesson 1 *Normal Distributions*

- Describe characteristics of a normal distribution
- Understand that the number of standard deviations from the mean is a measure of location
- When appropriate, fit a normal distribution to a set of data and use it to estimate percentages

Lesson 2 *Binomial Distributions*

- Use simulation to construct approximate binomial probability distributions
- Predict the shape of a binomial distribution
- Find and interpret expected values and standard deviations of binomial distributions
- Use a fitted normal distribution to estimate probabilities of events in binomial situations
- Use a random sample to make an inference about whether a specified proportion p is plausible as the population parameter

Lesson 3 *Statistical Process Control*

- Recognize when the mean and standard deviation change on a plot-over-time
- Use concepts of probability to create strategies for product testing
- Compute the probability of a false alarm on a set of readings, that is, the probability that a test will give an out-of-control signal for a process that is under control
- Understand the Central Limit Theorem and how it is applied to product testing

Unit 5: *Polynomial and Rational Functions*

Unit Objectives

- Recognize patterns in problem conditions and in data plots that can be described by polynomial and rational functions
- Write polynomial and rational function rules to describe patterns in graphs, numerical data, and problem conditions
- Use table, graph, or symbolic representations of polynomial and rational functions to answer questions about the situations they represent: (1) calculate y for a given x (i.e., evaluate functions); (2) find x for a given y (i.e., solve equations and inequalities); and (3) identify local max/min points and asymptotes
- Rewrite polynomial and rational expressions in equivalent forms by expanding or factoring, by combining like terms, and by removing common factors in numerator and denominator of rational expressions
- Add, subtract, and multiply polynomial and rational expressions and functions
- Extend understanding and skill in work with quadratic functions to include completing the square, interpreting vertex form, and proving the quadratic formula
- Recognize and calculate complex number solutions of quadratic equations

Lesson Objectives

Lesson 1 *Polynomial Expressions and Functions*

- Model problem situations using polynomial functions
- Identify patterns relating rules and graphs of polynomial functions—connecting polynomial degree to local maximum and local minimum values and zeroes
- Add, subtract, and multiply polynomials—connecting degrees of component polynomials to degrees of sums, differences, and products
- Find zeroes of polynomial functions and create polynomial functions with prescribed zeroes

Lesson 2 *Quadratic Polynomials*

- Express quadratic function rules in vertex form
- Use vertex form of quadratic expressions to solve quadratic equations and locate the vertex of parabolic graphs
- Use completing the square to prove the quadratic formula
- Use the quadratic formula to analyze solution possibilities for quadratic equations and indicate the rationale for extending the number system to include complex numbers

Lesson 3 *Rational Expressions and Functions*

- Create rational functions to model problem situations
- Analyze graphs of rational functions and their asymptotes
- Simplify rational expressions
- Add, subtract, multiply, and divide rational expressions

Unit 6: *Circles and Circular Functions*

Unit Objectives

- Discover, prove, and apply various properties of a line tangent to a circle, central angles, chords, arcs, and radii of a circle
- Discover, prove, and apply the Inscribed Angle Theorem and the property that inscribed angles that intercept the same or congruent arcs are congruent
- Analyze situations involving pulleys or sprockets to determine angular velocity and linear velocity
- Use sines and cosines to model aspects of circular motion and other periodic phenomena using both degrees and radians

Lesson Objectives

Lesson 1 *Circles and Their Properties*

- Discover and prove that a line tangent to a circle is perpendicular to the radius at the point of tangency and that the two tangent segments to a circle from the same external point are congruent
- Discover, prove, and apply the relationships among the measures of central angles, their chords, and their arcs
- Discover, prove, and apply the properties relating a radius, a chord, and the midpoint and perpendicular bisector of the chord
- Discover, prove, and apply the Inscribed Angle Theorem and the property that inscribed angles that intercept the same or congruent arcs are congruent

Lesson 2 *Circular Motion and Periodic Functions*

- Analyze situations involving pulleys or sprockets to study angular velocity and linear velocity
- Use sine and cosine functions to describe rotations of circular objects
- Use radian and degree measures to measure angles and rotations
- Relate the measure of a central angle of a circle to the length of the arc that the angle intercepts
- Define sine and cosine as functions of real numbers and analyze the resulting periodic graphs
- Use the sine and cosine functions to model periodic patterns of change in various physical phenomena

Unit 7: *Recursion and Iteration*

Unit Objectives

- Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change
- Formalize and consolidate previous study of *NOW-NEXT* rules, particularly through the use of subscript notation and the introduction of recursive formulas
- Understand and apply arithmetic and geometric sequences and series
- Understand and apply finite differences tables
- Explore function iteration and, in the process, informally introduce function composition
- Understand and apply recursive formulas, particularly combined recursive formulas of the form $A_n = rA_{n-1} + b$
- Review linear, exponential, and polynomial models from a recursive perspective

Lesson Objectives

Lesson 1 *Modeling Sequential Change*

- Use iteration and recursion to model real-world situations involving sequential change
- Understand the basic concepts of recursive formulas, particularly those of the form $A_n = A_{n-1} + b$
- Understand the effects of changing certain parameters on the long-term behavior of recursive formulas and the situations they model
- Use subscript notation and spreadsheet software to represent formulas that use the words *NOW* and *NEXT* and to take advantage of this notation and spreadsheet software to analyze recursive formulas more efficiently

Lesson 2 *A Recursive View of Functions*

- Understand arithmetic sequences and their connections to linear functions, using recursive formulas, function formulas, and applications
- Understand geometric sequences and their connections to exponential functions, using recursive formulas, function formulas, and applications
- Understand and apply arithmetic and geometric series (sums of sequences)
- Use finite differences tables to find function formulas for certain recursive formulas and to describe the connection between such tables and polynomial functions
- Use linear, exponential, and polynomial functions to model discrete situations

Lesson 3 *Iterating Functions*

- Iterate functions and describe the resulting patterns, the long-term behavior in particular
- Describe the connection between function iteration and recursive formulas
- Analyze long-term behavior when iterating linear functions, using graphical iteration, numerical iteration, and algebraic methods, including fixed point analysis and connections to slope

Unit 8: *Inverse Functions*

Unit Objectives

- Discover conditions that guarantee existence of inverse functions
- Develop strategies for finding rules for inverse functions
- Use inverse functions to solve problems of coding and decoding information
- Develop the definition and important properties of inverses for exponential functions
- Use logarithms to solve exponential equations
- Develop the definition and important properties of the inverse sine, cosine, and tangent functions
- Use inverse trigonometric functions to solve trigonometric equations and inequalities and the problems in which those equations and inequalities arise

Lesson Objectives

Lesson 1 *What Is An Inverse Function?*

- Solve problems involving direct and inverse variation
- Discover conditions that guarantee existence of an inverse for a given function
- Develop and use strategies for recognizing invertible functions from study of tables of values and/or graphs of those functions
- Develop and use strategies for finding rules of inverses for linear and power functions

Lesson 2 *Common Logarithms and Their Properties*

- Express a positive number as a power of 10
- Define and evaluate common logarithms
- Use logarithms to solve exponential equations
- Develop and use basic properties of the logarithmic function

Lesson 3 *Inverse Trigonometric Functions*

- Know and be able to use the definition of the inverse sine, inverse cosine, and inverse tangent functions
- Know and be able to use properties of the inverse sine, inverse cosine, and inverse tangent functions
- Use the inverse functions, to find one solution (when one exists) of $a \cdot f(bx) + c = d$, where $f(x)$ is the sine, cosine, or tangent
- Express the general solutions of a trigonometric equation in forms such as $x = k + 2\pi n$ or $x = k + 360^\circ n$ for any integer n
- Use trigonometric equations and their solutions to model and answer questions about periodic phenomena

Course 4

Unit 1: *Families of Functions*

Unit Objectives

- Review and extend properties of basic function families and their uses in mathematical modeling
- Develop strategies for finding rules of functions whose graphs are related by translation, reflection, stretching, and compressing to those of basic family members
- Apply enhanced function skills to build models of more complex relationships between variables
- Develop the definitions and important properties of arithmetic operations on functions
- Develop the definition, properties, and uses of function composition

Lesson Objectives

Lesson 1 ***Function Models Revisited***

- Review properties and applications of linear, exponential, power, sine, cosine, and inverse variation functions
- Develop a taxonomy of function rule, graph, and numerical patterns

Lesson 2 ***Customizing Models by Translation and Reflection***

- Discover the connections between rules of functions whose graphs are related by vertical translation, reflection across the x-axis, and horizontal translation
- Discover ways that maximum and minimum points, zeroes, and y-intercepts of two functions are related if their graphs are related by vertical translation, reflection across the x-axis, and horizontal translation
- Develop strategies for using the connections between graph transformations and function rules to develop models for relationships that are based on the core function families

Lesson 3 ***Customizing Models by Stretching and Compressing***

- Discover the connections between rules for functions that are related by horizontal and/or vertical stretching and compressing
- Develop strategies for adjusting basic sine and cosine functions to vary amplitude and period
- Discover relationships among maximum and minimum points, zeroes, and y-intercepts of functions whose graphs are related by vertical and/or horizontal stretching and compressing
- Use ideas of transformations of graphs to construct models of periodic variation

Lesson 4 ***Combining Functions***

- Develop the definitions and important properties of arithmetic operations on functions
- Discover meaningful ways to combine functions by arithmetic operations
- Develop understanding of function composition and skill in constructing rules for composite functions from rules of the component functions

Unit 2: *Vectors and Motion*

Unit Objectives

- Apply geometric concepts in modeling situations
- Describe and use the concept of vector in mathematical, scientific, and everyday situations
- Represent vectors geometrically and operate on geometric vectors
- Describe, represent, and use vector components and operations synthetically and analytically
- Investigate and justify general properties of vectors and vector operations
- Provide vector proofs of properties of triangles and parallelograms
- Use vector concepts to parametrically represent linear, projectile, circular, and elliptical motions in a plane
- Analyze motions using parametric models

Lesson Objectives

Lesson 1 *Modeling Linear Motion*

- Represent vectors as directed line segments with both direction and magnitude
- Describe and illustrate the scalar multiple and the opposite of a vector
- Describe and illustrate the components of a vector
- Add two vectors geometrically and by using components
- Model linear motions and forces with vectors
- Review and apply the Law of Sines and Law of Cosines to general triangles

Lesson 2 *Vectors and Parametric Equations*

- Describe and illustrate the relationship between the coordinates of the terminal point of a position vector and its components
- Explore and prove properties of scalar products and addition of vectors
- Describe and illustrate the dot product of two vectors, use it to compute the cosine of the angle formed by the vectors and to test whether or not those vectors are perpendicular
- Use vectors to prove properties of triangles and quadrilaterals
- Write parametric equations that model linear motion
- Simulate linear motion using technology

Lesson 3 *Modeling Nonlinear Motion*

- Write parametric equations to simulate projectile, circular, and elliptical motion
- Identify important forces affecting motion
- Use both radian and degree measurements to describe angular velocity
- Describe the location of a point on a rotating circle in terms of its initial location and angular velocity
- Simulate nonlinear motion using technology

Unit 3: *Algebraic Functions and Equations*

Unit Objectives

- Review and extend student skill in work with polynomials and polynomial functions to model graph patterns and conditions of quantitative relationships
- Develop basic properties of polynomial functions like zeroes, local max/min points, end behavior, and representation in standard, factored, and nested multiplication forms
- Develop understanding and skill in division of polynomials and the division algorithm
- Develop concepts of complex numbers and their representation in $a + bi$ form and as points/vectors on the coordinate grid
- Develop definitions and skill in operations on complex numbers
- Review and extend student understanding and skill in work with rational expressions and rational functions to model generalized inverse variation relationships
- Enhance student skill in analyzing rational functions and their graphs to identify domain, asymptotes, zeroes, and local max/min points
- Extend student skill in combining rational expressions by addition, subtraction, multiplication, and division and in solving equations involving rational expressions
- Develop skill in analyzing expressions and solving equations that involve radicals
- Reflect on generalizable strategies for solving algebraic problems by analysis of the forms of symbolic expressions and equations as well as by routine symbol manipulation

Lesson Objectives

Lesson 1 *Polynomial Function Models and Operations*

- Fit polynomial function models to data and graph patterns using problem conditions, statistical regression, and the method of undetermined coefficients
- Extend the relationship between standard, factored, and nested multiplication forms of polynomials
- Develop polynomial division and the division algorithm $p(x) = (x - k)q(x) + r(x)$
- Solve polynomial equations and inequalities

Lesson 2 *The Complex Number System*

- Develop understanding of the need for complex numbers to solve quadratic equations and the definition of the new numbers in the form $a + bi$, with a and b real numbers and $i = \sqrt{-1}$
- Use definitions of addition, subtraction, multiplication, and division of complex numbers to establish algebraic properties of complex number operations
- Develop geometric representation of complex numbers, including absolute value for magnitude, and the connection between complex number operations and basic geometric transformations.

Lesson 3 ***Rational Function Models and Operations***

- Write expressions for rules of rational functions that model patterns in experimental data, geometric curves, and problem conditions
- Identify asymptotes (horizontal, vertical, and oblique) for graphs of rational functions
- Review and extend skills in manipulating rational expressions into useful equivalent forms
- Solve rational equations and inequalities

Lesson 4 ***Algebraic Strategies***

- Write algebraic expressions for relationships that involve radicals
- Solve equations and inequalities involving radicals
- Review strategies for manipulating algebraic expressions and equations into equivalent forms and the justifications for those maneuvers
- Advance strategic thinking using symbol sense to analyze problem situations and their algebraic models both as an enhancement of and method for avoiding algebraic calculation

Unit 4: *Trigonometric Functions and Equations*

Unit Objectives

- Know and be able to use the definitions of the six trigonometric functions of an angle in standard position
- Derive and use the fundamental trigonometric identities
- Prove trigonometric identities
- Solve trigonometric equations
- Represent complex numbers geometrically
- Interpret multiplication and division of complex numbers geometrically
- Use De Moivre's Theorem to find powers and roots of complex numbers

Lesson Objectives

Lesson 1 *Reasoning with Trigonometric Functions*

- Know and be able to use the definitions of the six trigonometric functions
- Describe the graph and period of each trigonometric function
- Derive and use the fundamental trigonometric identities
- Develop strategies for proving trigonometric identities
- Derive and use the opposite-angle, cofunction, sum, difference, and double-angle identities for sine and cosine

Lesson 2 *Solving Trigonometric Equations*

- Solve linear and quadratic trigonometric equations
- Solve equations of the form $aT(bx + c) = d$, where T is a trigonometric function
- Express the general solutions of a trigonometric equation in forms such as $x = \alpha + 2\pi n$ or $x = \alpha + 360^\circ n$, for any integer n
- Use identities to transform trigonometric equations into more easily solved forms

Lesson 3 *The Geometry of Complex Numbers*

- Express a complex number in both standard and trigonometric forms
- Use complex number multiplication and division to size transform, rotate, or rotate and size transform the point or vector associated with a complex number
- Understand and use De Moivre's Theorem
- Determine all the nth roots of a complex number and represent them geometrically

Unit 5: *Exponential Functions, Logarithms, and Data Modeling*

Unit Objectives

- Understand e as the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$
- Use e^{rt} as an approximation for $\left(1 + \frac{r}{n}\right)^{nt}$
- Use functions of the form $y = Ae^{rt}$ to solve exponential growth and decay problems
- Show how any exponential function can be expressed in equivalent form using base e and how any logarithm can be expressed in equivalent form using base 10 or base e
- Use properties of exponents and logarithms to write algebraic expressions in equivalent forms and solve equations involving logs and exponents
- Use residual plots to evaluate the goodness of fit of linear regression equations
- Use logarithmic transformations of data to find linearized data patterns
- Use linear regression equations and back transformation (solving for y) to determine power and exponential functions that represent data patterns

Lesson Objectives

Lesson 1 *Exponents and Natural Logarithms*

- Understand e as the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$
- Use e^r as an approximation for $\left(1 + \frac{r}{n}\right)^n$ and e^{rt} as an approximation for $\left(1 + \frac{r}{n}\right)^{nt}$
- Use functions of the form $y = Ae^{rt}$ to solve exponential growth and decay problems
- Show how any exponential function can be expressed in equivalent form using base e and how any logarithm can be expressed in equivalent form using base 10 or base e
- Use properties of exponents and logarithms to write algebraic expressions in equivalent forms and solve equations involving logs and exponents

Lesson 2 *Linearization and Data Modeling*

- Use residual plots to assess the goodness of fit for linear models of data patterns
- Use logarithmic transformations of data to find linearized data patterns
- Use linear regression equations and back transformation (solving for y) to determine exponential and power functions that model data patterns

Unit 6: *Surfaces and Cross Sections*

Unit Objectives

- Represent three-dimensional objects and surfaces with contour lines or horizontal and vertical cross sections
- Interpret and describe three-dimensional surfaces or objects represented with contour diagrams
- Use the three-dimensional coordinate system to locate points and represent data, objects, and surfaces in space
- Identify and sketch graphs of conic sections represented algebraically and write equations matching graphs of conics
- Use information revealed by the form of an equation of a three-dimensional surface to visualize, characterize, and sketch the surface
- Identify and sketch surfaces of revolution and cylindrical surfaces

Lesson Objectives

Lesson 1 *Three-Dimensional Representations*

- Draw a contour diagram from appropriate data
- Describe and plot the location of a point in three dimensions using (x, y, z) coordinates
- Make a topographic profile that corresponds to a vertical cross section on a map displaying contour lines
- Identify cross sections of three-dimensional surfaces or objects
- Describe a surface given its contour diagram
- Sketch a three-dimensional object given a set of horizontal and vertical cross sections
- Identify and sketch graphs of conic sections given an equation in the form $ax^2 + by^2 + cx + dy + e = 0$
- Derive equations for circles given the center and radius
- Derive equations for parabolas given a focus and directrix
- Derive equations for ellipses and hyperbolas given the foci

Lesson 2 *Equations for Surfaces*

- Find the length of a segment that joins two points in space
- Find the coordinates of the midpoint of a line segment in space
- Describe the relationship between the coordinates of two points that are symmetric with respect to a coordinate plane or axis
- Determine the intercepts, the traces, the symmetry, and the coordinate-plane-parallel cross sections of a surface defined by an equation
- Recognize a plane as the graph of an equation $Ax + By + Cz = D$
- Sketch surfaces given their equations
- Recognize and sketch surfaces of revolution and cylindrical surfaces when given appropriate equations

Unit 7: *Concepts of Calculus*

Unit Objectives

- Develop the concept of instantaneous rate of change in a continuous variable and strategies for estimating those rates of change
- Define the derivative of a function at a point in its domain
- Connect the derivative of a function to local approximation of slope of its graph
- Develop derivative formulas for linear and quadratic functions
- Develop the connection between area under a rate function graph and accumulation of change
- Define the definite integral of a function and its application to problems

Lesson Objectives

Lesson 1 *Introduction to the Derivative*

- Develop the concept of instantaneous rate of change in a continuous variable and strategies for estimating those rates of change
- Define the derivative of a function at a point in its domain
- Connect the derivative of a function to local approximation of slope of its graph
- Develop derivatives formulas for linear and quadratic functions

Lesson 2 *Introduction to the Definite Integral*

- Develop understanding of the connection between cumulative change and area bounded by a rate of change graph
- Develop beginning understanding of the ways that areas (and thus definite integrals) can be approximated by Riemann sums and the effects of refining the approximation by letting $\Delta x \rightarrow 0$

Unit 8: *Counting Methods and Induction*

Unit Objectives

- Develop the skill of careful counting in a variety of contexts
- Understand and apply a variety of counting techniques, including the Multiplication Principle of Counting, the Addition Principle of Counting, counting trees, and systematic lists
- Understand and apply the issues of order and repetition when counting the number of possible choices from a collection
- Solve counting problems involving combinations and permutations
- Understand and apply the Binomial Theorem
- Understand and apply connections among combinations, the Binomial Theorem, and Pascal's triangle
- Apply counting methods to probability situations in which all outcomes are equally likely
- Extend understanding of and apply the General Multiplication Rule for Probability
- Develop the skill of combinatorial reasoning, including use in proofs
- Understand and carry out proofs by mathematical induction
- Understand and carry out proofs using indirect reasoning and the Least Number Principle

Lesson Objectives

Lesson 1 *Systematic Counting*

- Develop the skill of systematic counting by thinking carefully about the number of possibilities in a variety of contexts
- Understand and apply basic counting strategies, such as making tree diagrams, making systematic lists, and using the Multiplication Principle of Counting and the Addition Principle of Counting
- Understand the importance of counting in many real-world contexts

Lesson 2 *Order and Repetition*

- Understand and apply order and repetition in counting problems
- Create, analyze, apply, and make sense of the formulas for counting permutations and combinations
- Solve counting problems by applying several methods and concepts in any given problem
- Understand, apply, and connect three basic counting problems—counting selections from a collection of objects (the four order and repetition problem types), counting the outcomes from a sequence of tasks (the Multiplication Principle of Counting), and counting the outcomes from a union (the Addition Principle of Counting).
- Understand the importance of counting in many real-world contexts

Lesson 3 *Counting Throughout Mathematics*

- Apply counting methods to probability situations in which all outcomes are equally likely
- Review and apply the General Multiplication Rule for Probability; in particular, use the Multiplication Principle of Counting, permutations, and combinations
- Understand and apply the Binomial Theorem
- Understand and apply connections among combinations, the Binomial Theorem, and Pascal's triangle
- Develop the skill of combinatorial reasoning, including its use in proofs

Lesson 4 *Proof by Mathematical Induction*

Note: The material in this lesson is important for students planning to major in mathematics or computer science in college. However, the lesson can be omitted for other students.

- Understand the Principle of Mathematical Induction
- Develop skill in proving statements by mathematical induction
- Understand the Least Number Principle and proofs using this principle along with indirect reasoning