PRINT AND DIGITAL CURRICULUM DESIGN IN THE U.S.: THE CASE OF TRANSITION TO COLLEGE MATHEMATICS AND STATISTICS

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This paper describes a four-year project in the U.S., funded by the National Science Foundation, to design, develop, and evaluate Transition to College Mathematics and Statistics (TCMS), a senior-level course to help meet the diverse quantitative needs of students whose intended undergraduate programs do not require calculus. For students intending to enroll in non-calculus-based undergraduate programs, many schools in the U.S. have little to offer as a transition to college-level mathematics and statistics other than precalculus or narrow Advanced Placement courses. Consequently, many students opt out of mathematics their senior year or study mathematics that is inappropriate for their undergraduate and career aspirations. TCMS focuses on broadly useful topics, including mathematical modeling, data analysis and inference, informatics, financial mathematics, decision-making under constraints, mathematical visualization and representations, and important mathematical habits of mind.

INTRODUCTION

Transition to College Mathematics and Statistics (TCMS) (Hirsch et al., 2015) is a problem-based, inquiry-oriented, and technology-rich fourth-year high school mathematics course. It was developed to help ensure student success in college and careers in an increasingly technological, information-laden, and data-driven global society. TCMS was specifically designed for the large numbers of non-STEM (science, technology, engineering, and mathematics) students whose undergraduate programs of study do not require calculus. The top 10 undergraduate majors in the U.S., as reported in the Princeton Review (Top 10 College Majors, 2014), are shown in Fig. 1.

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<th>Top 10 Undergraduate Majors in the U.S.</th>
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<tr>
<td>1. Business Administration and Management/Commerce</td>
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<td>2. Psychology</td>
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<td>3. Nursing</td>
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<tr>
<td>4. Biology/Biological Sciences</td>
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<td>5. Education</td>
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<td>6. English Language and Literature</td>
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<td>7. Economics</td>
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<td>8. Communications Studies/Speech Communication and Rhetoric</td>
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<td>9. Political Science and Government</td>
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<td>10. Computer and Information Sciences</td>
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Figure 1: The case for an alternative to precalculus as a fourth-year course.
Students intending to complete one of these majors or other non-STEM majors in college are often ill-prepared by the lack of appropriate fourth-year high school mathematics courses. Research has repeatedly shown that students who are not enrolled in an appropriate mathematics course their senior year are much more likely to be placed in a remedial (non-credit bearing) course in college (cf. Key & O’Malley, 2014). It is in this context that TCMS was designed, developed, and evaluated.

*Transition to College Mathematics and Statistics* is intended as a fourth-year capstone course for students who have successfully completed a conventional single-subject sequence of algebra, geometry, and advanced algebra or a three-year unified (international-like) mathematics sequence. The course has been carefully field tested in wide range of high schools with students using conventional mathematics curricula and with students using a unified mathematics program.

**DESIGN FEATURES**

Key themes and instructional features as outlined below have been informed by research on student learning (cf. National Research Council, 2005; NCTM, 2014), by recommendations from client disciplines on the focus of undergraduate mathematics and statistics courses for non-STEM students (cf. Ganter & Barker, 2004), and lessons learned over the last 20+ years from iterative cycles of design, development, field testing, revision, and external efficacy studies of *Core-Plus Mathematics*, a four-year comprehensive unified curriculum for high schools (Fey & Hirsch, 2007, www.cpmonline.org).

**Balanced Content**—*Transition to College Mathematics and Statistics* reviews and extends students’ understanding of important and broadly useful concepts and methods from algebra and functions, statistics and probability, discrete mathematics, and geometry. These branches of mathematics are connected by the central theme of modeling our world and by mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, exploiting use of multiple representations, providing convincing explanations, and a disposition toward strategic use of technological tools. See fig. 2 for a list of TCMS units, goals, and key topics.

**Flexibility**—TCMS units are focused, coherent, and, in most cases, generally self-contained with attention to content prerequisites provided by “Just-in-Time” review and bridging tasks in lesson homework sets. The course has been organized to be as flexible as possible. The organization permits teachers to tailor courses that best meet the needs and interests of their students. For example, some teachers choose to use the unit on *Mathematics of Democratic Decision-Making* as the second or third unit of the course to parallel state or national elections in the U.S.

**Mathematical Modeling**—TCMS emphasizes mathematical modeling (including the processes of problem formulation, data collection, representation, simulation, interpretation, and prediction) and its cyclic nature. Problem situations and phenomena modeled in this course involve discrete and continuous variables and entail deterministic as well as stochastic processes.
**Transition to College Mathematics and Statistics**

| Unit 1 | **Interpreting Categorical Data** | develops student understanding of two-way frequency tables, conditional probability and independence, and using data from a randomized experiment to compare two treatments.  
*Topics include* two-way tables, graphical representations, comparison of proportions including absolute risk reduction and relative risk, characteristics and terminology of well-designed experiments, expected frequency, chi-square test of homogeneity, statistical significance. |
|---|---|---|
| Unit 2 | **Functions Modeling Change** | extends student understanding of linear, exponential, quadratic, power, circular, and logarithmic functions to model quantitative relationships and data patterns whose graphs are transformations of basic patterns.  
*Topics include* linear, exponential, quadratic, power, circular, and base-10 logarithmic functions; mathematical modeling; translation, reflection, stretching, and compressing of graphs with connections to symbolic forms of corresponding function rules. |
| Unit 3 | **Counting Methods** | extends student ability to count systematically and solve enumeration problems using permutations and combinations.  
*Topics include* systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal’s triangle, combinatorial reasoning; and the general multiplication rule for probability. |
| Unit 4 | **Mathematics of Financial Decision-Making** | extends student facility with the use of linear, exponential, and logarithmic functions, expressions, and equations in representing and reasoning about quantitative relationships, especially those involving financial mathematical models.  
*Topics include* forms of investment, simple and compound interest, future value of an increasing annuity, comparing investment options, continuous compounding and natural logarithms; amortization of loans and mortgages, present value of a decreasing annuity, and comparing auto loan and lease options. |
| Unit 5 | **Binomial Distributions and Statistical Inference** | develops student understanding of the rules of probability; binomial distributions; expected value; testing a model; simulation; making inferences about the population based on a random sample; margin of error; and comparison of sample surveys, experiments, and observational studies and how randomization relates to each.  
*Topics include* review of basic rules and vocabulary of probability (addition and multiplication rules, independent events, mutually exclusive events); binomial probability formula; expected value; statistical significance and P-value; design of sample surveys including random sampling and stratified random sampling; response bias; sample selection bias; sampling distribution; variability in sampling and sampling error; margin of error; and confidence interval. |
| Unit 6 | **Informatics** | develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency.  
*Topics include* elementary set theory and logic; modular arithmetic and number theory; secret codes, symmetric-key and public-key cryptosystems; error-detecting codes (including ZIP, UPC, and ISBN) and error-correcting codes (including Hamming distance); and trees and Huffman coding. |
| Unit 7 | **Spatial Visualization and Representations** | extends student ability to visualize and represent three-dimensional shapes using contour diagrams, cross sections, and relief maps; to use coordinate methods for representing and analyzing three-dimensional shapes and their properties; and to use graphical and algebraic reasoning to solve systems of linear equations and inequalities in three variables and linear programming problems.  
*Topics include* using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; three-dimensional rectangular coordinate system; sketching planes using traces, intercepts, and cross sections derived from algebraic representations; systems of linear equations and inequalities in three variables; and linear programming. |
| Unit 8 | **Mathematics of Democratic Decision-Making** | develops student understanding of the mathematical concepts and methods useful in making decisions in a democratic society, as related to voting and fair division.  
*Topics include* preferential voting and associated vote-analysis methods such as majority, plurality, runoff, points-for-preferences (Borda method), pairwise-comparison (Condorcet method), and Arrow’s theorem; weighted voting, including weight and power of a vote and the Banzhaf power index; and fair division techniques, including apportionment methods. |

Figure 2: TCMS units are amenable to alternative sequencing.

**Technology**—**TCMS-Tools®,** a suite of curriculum-embedded software tools, including a spreadsheet; a CAS; dynamic geometry; data analysis tools; discrete modeling tools; and specialized apps (e.g., linear programming, binomial distributions, contour diagrams, and code
encryption and decryption), co-developed and field tested with the curriculum (see figs. 3 and 4) and graphing calculators are assumed and strategically used throughout the course.

Active Learning—The instructional materials are designed to promote cycles of active learning and teaching centered around a teacher-led launch of a problem-situation for students to think about (that sets the context for the student work to follow, provides student motivation, and reveals important information about students’ prior knowledge), followed by collaborative team investigations of related applied problem situations, and capped by a teacher-led whole-class summarizing activity that leads to analysis, abstraction, and further application of underlying mathematical ideas and principles. Throughout students are actively engaged in modeling and in exploring, conjecturing, verifying, generalizing, applying, evaluating, and communicating mathematical ideas that emerge.

Multi-dimensional Assessment—Comprehensive assessment of student understanding and progress through both curriculum-embedded formative assessment opportunities and summative assessment tasks support instruction and enable monitoring and evaluation of each student’s performance in terms of mathematical practices, content, and dispositions.

KEY EVALUATION FINDINGS

The following findings are based on multiple beginning and end-of-year measures in six high schools located in Colorado, Kentucky, Michigan, New York, and Texas:

- Three schools in which students had completed a single-subject sequence of algebra, geometry, and advanced algebra using programs from different publishers;
- Three schools in which students had completed a three-year unified mathematics program.

TCMS End-of-Year Attitude Survey

Among the major findings revealed by the end-of-year attitude survey:

Students across all six field-test schools generally found the statistics and discrete mathematics units to be most interesting, noted the mathematics in each unit was reasonable to understand, and found the contexts engaging and something they could relate to.

Students from schools using conventional single-subject courses couched many of their comments in terms of the TCMS real-world problems in contrast to the non-context experiences they generally encountered in their previous courses.
ITED Highest Level Quantitative Thinking Test

The ITED Quantitative Thinking Test for end of grade 12 consists of multiple forms of 40 items focusing on thinking, reasoning, and problem-solving skills. All schools showed pre-post gains greater than expected in terms of national norms. One school using the unified program made significant gains at the $p = .05$ level, as did one school using a conventional curriculum. Across all schools, gains were particularly notable for students in the third and fourth quartiles.

Analysis of student placement and performance in their first college mathematics or statistics course is in process.

DIGITAL VERSION: A DESIGN EXPERIMENT

In addition to design, development, and classroom trials of the TCMS print units, the project also conducted a design experiment (Brown 1992; Collins 1992, 1999; Design-based Research Collective) to create a digital version of the Binomial Distributions and Statistical Inference unit that could serve as a design prototype for units in the digital version of TCMS. Our goal was to iteratively design a “deeply digital” (Dorsey, 2011) unit that went beyond current digital curricula that Choppin et al (2014) found to be primarily individualized learning/intervention programs and digitized versions of conventional textbooks. In particular, we wanted to take TCMS content and take the properties of digital media to deliver the content in a completely new way that is much stronger for learning (cf. Roschelle, 2011).

The digital version of the Binomial Distributions and Statistical Inference unit (see Figure 5 for Unit opener) extends the features of the print version to provide (1) use of embedded video for lesson launches and selected applications; (2) flexibility in how users engage with the materials such as adjusting sound, changing the size of the text, controlling graphics; (3) learner-controlled scaffolding of complex problems; (4) greater accessibility to outside tools such as text-to-speech software and translation software; and (5) a network of embedded hyperlinks, allowing immediate access to specific sections within the text, mathematical tools such as TCMS-Tools, Internet Web sites, and other resources.

As shown in Figure 6, the digitally based lessons are accessed in a digital shell consisting of four components. The Resource Navigation component allows students and the teacher to navigate to the unit home screen, launch mathematical and statistical software, activate text-to-speech software and the glossary of key terms, and refresh the digital collaborative notebooks. The Investigation Navigation component most resembles what students and teachers might experience using conventional print materials. Students and the teacher can navigate to different components of the lesson and activate accessibility features. The Embedded Notes component allows students and the teacher to copy and paste a website address to display their digital collaborative notebooks. The
Embedded Resource component provides access to launch the mathematical and statistical software, glossary of key terms, text-to-speech, and embedded audio and video clips.

Figure 6: Digital shell.

Figure 7 illustrates design of the embedded digital collaborative notebook, a problem linked to learner-controlled scaffolding, and embedded mathematical/statistical tools. For more details on the design features of this prototype unit, see Edson (2014, 2016).

Figure 7: Learner-controlled scaffolding, digital collaborative notebook, and embedded mathematical/statistical software.
FINDINGS RELATED TO DESIGN FEATURE USE

Digital Shell—Students interacted with other students and the teacher while synchronously interacting with the mathematical and statistical software, accessibility features, open-ended problems linked to learner-controlled scaffolding, embedded audio and video clips, and digital collaborative notebooks.

Mathematical and Statistical Software—Students embedded TCMS-Tools screenshots into their digital collaborative notebooks as they used spreadsheets, graphs, and computer algebra systems (CAS) to solve binomial distributions problems.

Accessibility Features—Students used the glossary of key terms and scroll-over definitions to recall definitions and examples. They solved problems together in their groups using the ‘problem on its own tab’ feature. Students accessed embedded information using the hyperlinks. Students indicated that the embedded hyperlinks, glossary of key terms, scroll-over definitions, text-to-speech, print feature, adjustment of text size, and ‘problem on its own tab’ are valuable features.

Open-ended Problems Linked to Learner-controlled Scaffolding—Students solved complex and more open-ended problems. They took ownership of their learning using the learner-controlled scaffolding without teacher intervention. Students self-assessed their own thinking and the work of their group and initiated group discussions using the learner-controlled scaffolding.

Embedded Audio and Video Clips—Students accessed the embedded audio and video clips. They related the problem situations to the embedded audio and video clips. Furthermore, students indicated that they accessed the audio and video clips because they were curious to learn more about problem situations.

Digital Collaborative Notebooks—Students contributed to, revised, reflected on, and assessed their own thinking and the thinking of their team mates. They used templates to help organize their work in a synchronous environment shared with their team mates and their teacher. They incorporated tables, sketches, and photos into their digital collaborative notebooks when solving problems. Students stayed together with their team mates before moving to additional investigation problems. Students accessed their completed group work on their digital collaborative notebooks outside of class to help solve individual homework problems.

For further details on iterative classroom trials of this prototype unit, see Lemmen (2016).

DESIGN EXPERIMENT SUMMARY

Students solved complex and more open-ended problems. They took ownership of their learning using the learner-controlled scaffolding without teacher intervention. Students self-assessed their own thinking and the work of their team members. They initiated team discussions related to the use of the learner-controlled scaffolding. For teachers, the learner-controlled scaffolding represents a pedagogical tool for optimizing cognitive demand while providing multiple access points for students. If and when students asked for help, the nature of student questions focused more on understanding the underlying mathematics/statistics of the problem situation rather than on getting started.
Hirsch

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References


